Nomination form for the 2021 Nishina Asia Award

Candidate (name, affiliation, curriculum vitae including the date of the degree of Ph.D., nationality, address, email and telephone)

Name: Wei Song Affiliation: Professor at Tsinghua University, Beijing, China

Date of Birth: April 14, 1983 Nationality: Chinese

Email: wsong2014@mail.tsinghua.edu.cn Phone: +86 13641040458

Address: Jing Zhai 309, Tsinghua University, Hai Dian District, Beijing, 100084, China

Date of the degree of Ph.D: July 4, 2009 (Thesis Title: Quantum Gravity in Three Dimensions)

Biography:

2000-2004 : Bachelor of Science, Nanjing University, Nanjing, China

2004-2009: PhD, The Institute of Theoretical Physics, Chinese Academy of Sciences (Advisor: Prof. Miao Li)

2009-2013: Junior Fellow, Harvard University

2013-2014: Postdoc Associate, Princeton University

2014-present: Faculty Member, Tsinghua University

Citation for the Award (within 30 words)

Her outstanding contributions to holography in warped anti-de Sitter spaces

Description of the work

Prof. Wei Song is one of a few extremely outstanding Asian young researchers in high energy theory and she is the best female young researcher in string theory not only in Asia but also in the world, as far as I know. She has published 43 papers and has earned totally more than 2800 citations until now (in this letter I always refer to INSPIRE database). Three of her papers earn more than 250 citations. With these remarkable research achievements, she has now been a very responsible leader of the Chinese string theory community as a full professor at Tsinghua University. For example, she was virtually the main organizer of the strings 2016 conference held in Tsinghua University. Strings conference is the annual largest event in string theory community.

Prof. Wei Song has contributed to string theory tremendously in the subject of holography or gauge/gravity correspondence. The holography in string theory tells us that a gravitational theory (or string theory) in d+1 dimensional spacetime is equivalent to a d dimensional non-gravitational theory such as quantum field theories. The most standard example of holography is AdS/CFT, where gravitational theories on anti-de Sitter (AdS) spaces are equivalent to conformal field theories (CFTs). In particular, Prof. Wei Song is very famous for pioneering a deformed version of AdS/CFT where a part of conformal symmetry (a half of Virasoro symmetries) was broken, which has numerous important applications.

First of all, she wrote extremely famous paper of Kerr/CFT correspondence based on collaborations with Prof. Strominger and others (ref.[1]). This paper argues that Kerr black holes are equivalent to certain two dimensional chiral CFTs and they gave a microscopic derivation of

Kerr black hole entropy. This paper, which has been cited more than 670 times, gave a huge impact on string theory because it extends the validity of the AdS/CFT to astrophysical objects.

The idea of breaking a half of Virasoro symmetry expands holographic dualities further and she discovered a new class of holography, called warped AdS (WAdS) duality, with Prof. Strominger and others (ref.[2]). They found that a gravity on warped AdS is equivalent to the warped conformal field theories (WCFT), which is a novel non-relativistic quantum field theory with SL(2;R)×U(1) symmetry as the global symmetry. This duality has attracted much attention and this paper has been cited more than 300 times.

The warped AdS holography, which Prof. Wei Song pioneered, has recently attracted much attention in the context of integrable irrelevant deformations of CFTs. By combining with the recently technique, she and her post-doc recently identified the precise WCFT which is equivalent to string theory in WAdS (ref.[3]). They confirmed that the spectrum and entropy perfectly agree between WAdS and WCFT. She has often been invited to important conferences in this field. For example, she was a guest speaker in the KITP conference: Geometry from the Quantum, at Santa Barbara in Jan.2020 and in Solvay Workshop on Holography, Brussels in May 2018. Obviously, Prof. Wei Song is the top researcher in the worlds on this important aspect of holography.

Due to thses reasons, I recommend Prof. Wei Song for Nishina Asia Award in my strongest terms.

Key references (up to 3 key publications*)

1] M. Guica, T. Hartman, W. Song and A. Strominger, ``The Kerr/CFT Correspondence,''
Phys. Rev. D 80 (2009) 124008 [arXiv:0809.4266 [hep-th]]. [INSPIRE citation 674 times]

[2] D. Anninos, W. Li, M. Padi, W. Song and A. Strominger, ``Warped AdS(3) Black Holes,''
JHEP 0903 (2009) 130 [arXiv:0807.3040 [hep-th]]. [INSPIRE citation 301 times]

[3] L. Apolo and W. Song,``Strings on warped AdS3 via TJbar deformations,''
JHEP 1810 (2018) 165 [arXiv:1806.10127 [hep-th]]. [INSPIRE citation 56 times]

*) Copy of one most significant publication should be attached.

Nominator (name, affiliation, email, telephone and relation to the candidate)

Name: Tadashi Takayanagi

Affiliation: Professor, Yukawa Institute for Theoretical Physics, Kyoto University

Email: takayana@yukawa.kyoto-u.ac.jp

Phone: 090-3041-5054

Relation to the candidate: The nominator has been working on subjects closely related to those of candidates. The nominator has often discussed physics with the candidates in various conferences and workshops abroad. Last year the nominator invited the candidate to a conference hosted in YITP, Kyoto, ``String Theory and Quantum Information", where she gave a beautiful plenary invited talk on ref. [3] and its developments in the presence of more than 200 participants.

Tadashi Takayanagi

Signature Date Jan.30th, 2021

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The Kerr/CFT correspondence

Monica Guica,[1] Thomas Hartman,[2] Wei Song,[2,*] and Andrew Strominger[2]
[1]*Laboratoire de Physique Théorique et Hautes Energies, Université Pierre et Marie Curie-Paris 6; CNRS Boîte 126, 4 Pl. Jussieu, 75252 Paris Cedex 05, France*
[2]*Center for the Fundamental Laws of Nature, Jefferson Physical Laboratory, Harvard University, Cambridge, Massachusetts 02138, USA*
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Quantum gravity in the region very near the horizon of an extreme Kerr black hole (whose angular momentum and mass are related by $J = GM^2$) is considered. It is shown that consistent boundary conditions exist, for which the asymptotic symmetry generators form one copy of the Virasoro algebra with central charge $c_L = \frac{12J}{\hbar}$. This implies that the near-horizon quantum states can be identified with those of (a chiral half of) a two-dimensional conformal field theory (CFT). Moreover, in the extreme limit, the Frolov-Thorne vacuum state reduces to a thermal density matrix with dimensionless temperature $T_L = \frac{1}{2\pi}$ and conjugate energy given by the zero mode generator, L_0, of the Virasoro algebra. Assuming unitarity, the Cardy formula then gives a microscopic entropy $S_{\mathrm{micro}} = \frac{2\pi J}{\hbar}$ for the CFT, which reproduces the macroscopic Bekenstein-Hawking entropy $S_{\mathrm{macro}} = \frac{\mathrm{Area}}{4\hbar G}$. The results apply to any consistent unitary quantum theory of gravity with a Kerr solution. We accordingly conjecture that extreme Kerr black holes are holographically dual to a chiral two-dimensional conformal field theory with central charge $c_L = \frac{12J}{\hbar}$, and, in particular, that the near-extreme black hole GRS 1915+105 is approximately dual to a CFT with $c_L \sim 2 \times 10^{79}$.

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I. INTRODUCTION

One of the deepest discoveries in modern theoretical physics is that of holographic dualities, which relate a quantum theory of gravity to a quantum field theory without gravity in fewer dimensions. These dualities become especially powerful when combined with string theory [1]. It is an occasional misconception, however, that the existence of holographic dualities is contingent on the validity of string theory. This is not the case. For example, the demonstration [2] that any consistent theory of quantum gravity on three-dimensional anti-de Sitter space (AdS_3) is holographically dual to a two-dimensional conformal field theory (CFT) did not invoke string theory. When holographic duality was used to find the microscopic origin of the Bekenstein-Hawking entropy for a class of black holes, the construction at first appeared to depend heavily on details of string theory [3]. However, it was later understood [4] to apply to essentially any consistent, unitary quantum theory of gravity containing the black holes as classical solutions. In the last few years we are beginning to see interesting applications of holographic duality outside of string theory in nuclear [5–7], condensed matter [8–10], and atomic [11,12] physics.

Oddly, the rich ideas surrounding holographic dualities so far have not been successfully applied to the enigmatic objects which largely inspired their original discovery—the Schwarzschild or Kerr black holes we actually observe in the sky.[1] In this paper we attempt to fill this gap by arguing, in the spirit of [2,4], that extreme Kerr black holes are holographically dual to a chiral CFT in two dimensions. An extreme Kerr black hole is one for which the angular momentum J saturates the bound $J \leq GM^2$. More angular momentum with the same mass M leads to a violation of cosmic censorship. Nearly extreme black holes have been seen in the sky. For example GRS 1915+105, with mass $M \sim 14M_{\mathrm{sun}}$, has $J/GM^2 > 0.98$ [13], and corrections to the dual CFT representation of GRS 1915+105 should be correspondingly suppressed. In addition, at extremality the ISCO (the innermost stable circular orbit on the accretion disc) coincides with the event horizon, so near extremality the ISCO is within the near-horizon region. Therefore the observed emissions from the ISCO should be well-described by the dual CFT.[2] It is our hope that the rich experimental [13,16] and theoretical [17] literature on Kerr black holes can be illuminated by the dual CFT description.

[*]On leave from the Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China.

[1]The successes so far have mainly concerned black holes with large amounts of charge and in dimensions other than four.

[2]In [14,15] greybody scattering factors for various black holes were computed using the dual CFT picture, and found to agree with those computed by conventional methods. Computations of this type may also be possible for Kerr, and generalized to the context of accretion discs.

Our argument that Kerr is dual to a CFT parallels the general one given by Brown and Henneaux [2] for AdS_3, except that we replace AdS_3 with the NHEK (near-horizon extreme Kerr) geometry found by Bardeen and Horowitz [18] via a near-horizon limiting procedure.[3] Despite having different dimensions, the spaces bear some resemblance: a slice of NHEK at a particular fixed polar angle is a discrete quotient of AdS_3. We first carefully specify boundary conditions at the asymptotic infinity of NHEK (which is where, before taking the near-horizon limit, it is joined to Minkowski space in the full Kerr solution) and demonstrate their consistency. We then show that, given these boundary conditions, the so-called asymptotic symmetry group (ASG) is one copy of the conformal group and furthermore has a central charge $c_L = \frac{12J}{\hbar}$. Hence extreme Kerr, with the given boundary conditions, is dual to a chiral CFT.[4]

While this very general analysis gives the central charge of the dual CFT, it tells us little else about the detailed structure of the CFT. For that to be determined we would need an ultraviolet completion (for example string theory) of quantum gravity on the Kerr background. However the information about the central charge, together with the assumption of unitarity, turns out to be exactly enough to compute the extreme Kerr entropy by counting quantum microstates, as in [4]. An analysis of the extreme limit of the Frolov-Thorne vacuum, which generalizes the Hartle-Hawking vacuum for Schwarzschild to Kerr, shows that the CFT must be at temperature $T_L = \frac{1}{2\pi}$. We then apply the thermodynamic Cardy formula relating the microscopic entropy of a unitary CFT to its temperature and central charge. The resulting entropy agrees exactly with the macroscopic Bekenstein-Hawking area-entropy law, providing corroboration for our proposal that extreme Kerr is dual to a two-dimensional chiral CFT.

The fact that we encounter only a chiral half of a CFT ultimately derives from the fact that at extremality the rotational velocity of the Kerr horizon becomes the speed of light. Hence both edges of the forward light cone coincide as the horizon is approached and force all physical excitations (such as the edge of the accretion disc), which must lie between the edges of light cone, to spin around chirally with the black hole. Away from extremality this is no longer the case and we may expect to encounter a nonchiral CFT. This very interesting but difficult problem will not be considered herein.

We wish to stress that, while mere consistency imposes very strong constraints, we have not analyzed all possibilities and have not shown that our near-horizon boundary conditions are the unique consistent choice for studying extreme Kerr. While we did not find any other consistent and nontrivial choices, our search was not exhaustive, and there may well be others with different consequences. Ultimately, the appropriate boundary conditions should be determined from the physical question. We do suspect that weaker or different boundary conditions will be needed for the just-mentioned problem of near-extremal excitations. These issues remain for future work.

Section II reviews the Kerr geometry and Sec. III its near-horizon limit. In Sec. IV we review the notion of an ASG. Our boundary conditions are specified in Sec. V, and the generators L_n of the corresponding ASG are shown to form a Virasoro algebra in Sec. VI. The central charge is computed in Sec. VII. In Sec. VIII we take the limit of the Frolov-Thorne vacuum for Kerr, and show that it yields a thermal state with temperature $\frac{1}{2\pi}$. In the concluding section we microscopically compute the entropy for extreme Kerr from the Cardy formula and find that it reproduces the macroscopic Bekenstein-Hawking area law. Some technical points are relegated to two appendices.

Previous work on a dual description of Kerr, some in the context of string theory, includes [14,19–27].

II. KERR REVIEW

The Kerr metric [28,29] is the general rotating black hole solution of the four-dimensional vacuum Einstein equations. In Boyer-Lindquist coordinates it is

$$ds^2 = -\frac{\Delta}{\rho^2}(d\hat{t} - a\sin^2\theta d\hat{\phi})^2 + \frac{\sin^2\theta}{\rho^2}((\hat{r}^2 + a^2)d\hat{\phi} - ad\hat{t})^2 + \frac{\rho^2}{\Delta}d\hat{r}^2 + \rho^2 d\theta^2 \tag{2.1}$$

$$\Delta \equiv \hat{r}^2 - 2M\hat{r} + a^2, \qquad \rho^2 \equiv \hat{r}^2 + a^2\cos^2\theta, \tag{2.2}$$

$$a \equiv \frac{GJ}{M}, \qquad M \equiv GM_{\mathrm{ADM}}. \tag{2.3}$$

It is labeled by two parameters: the angular momentum J and the geometric mass M. In order to simplify the formulas, but at the risk of some confusion, in the above and hereafter we have rescaled M by a factor of G relative to the abstract and introduction. The solution has naked singularities unless J lies in the parameter range

$$\frac{-M^2}{G} \leq J \leq \frac{M^2}{G}. \tag{2.4}$$

Of course, quantum mechanically J is quantized

[3] In this procedure the asymptotically flat region, whose excitations we do not regard as part of the black hole itself, is excised and one is left only with the portion of the spacetime neighboring the black hole horizon.

[4] We do not have an argument for modular invariance and are not distinguishing here between the chiral sector of a nonchiral CFT and a CFT with only a chiral sector. It is interesting to note however that a necessary condition for the partition function of the latter to be modular invariant up to a sign, accounting for the presence of fermions, given $c = 12J/\hbar$ is precisely that $J/\hbar$ is half-integral.

$$J = \hbar j \tag{2.5}$$

for some half integer j. There is an event horizon at

$$r_+ = M + \sqrt{M^2 - a^2}. \tag{2.6}$$

The Hawking temperature, surface gravity and angular velocity of the horizon are

$$T_H = \frac{\hbar\kappa}{2\pi} = \frac{\hbar(r_+ - M)}{4\pi M r_+}, \tag{2.7}$$

$$\Omega_H = \frac{a}{2Mr_+}. \tag{2.8}$$

These are related by the first law to the Bekenstein-Hawking entropy [30,31]

$$S_{\rm BH} = \frac{\text{Area}}{4\hbar G} = \frac{2\pi M r_+}{\hbar G}. \tag{2.9}$$

We are primarily interested in the so-called extreme Kerr, which carries the maximum allowed angular momentum

$$J = \frac{M^2}{G}. \tag{2.10}$$

Extreme Kerr has zero Hawking temperature but a nonzero entropy

$$S_{\rm BH} = \frac{2\pi J}{\hbar} \tag{2.11}$$

Our goal is to explain this number as the logarithm of the number of quantum microstates of Kerr.

III. THE NHEK GEOMETRY

We wish to study the region very near the extreme Kerr horizon at $\hat{r} = M$. In order to do so, following Bardeen and Horowitz [18] we define new (dimensionless) coordinates

$$t = \frac{\lambda \hat{t}}{2M}, \qquad y = \frac{\lambda M}{\hat{r} - M}, \qquad \phi = \hat{\phi} - \frac{\hat{t}}{2M} \tag{3.1}$$

and take $\lambda \to 0$ keeping (t, y, ϕ, θ) fixed. The result is the near-horizon extreme Kerr or "NHEK" geometry in Poincaré-type coordinates

$$ds^2 = 2GJ\Omega^2\left(\frac{-dt^2 + dy^2}{y^2} + d\theta^2 + \Lambda^2\left(d\phi + \frac{dt}{y}\right)^2\right) \tag{3.2}$$

where

$$\Omega^2 \equiv \frac{1 + \cos^2\theta}{2}, \qquad \Lambda \equiv \frac{2\sin\theta}{1 + \cos^2\theta}, \tag{3.3}$$

$\phi \sim \phi + 2\pi$ and $0 \le \theta \le \pi$. The NHEK geometry is not asymptotically flat. Note that the angular momentum affects only the overall scale of the geometry.

The coordinates (3.2) cover only part of the NHEK geometry. Global coordinates (r, τ, φ) are given by (for a discussion of global properties see [18])

$$y = (\cos\tau\sqrt{1 + r^2} + r)^{-1}, \tag{3.4}$$

$$t = y \sin\tau\sqrt{1 + r^2}, \tag{3.5}$$

$$\phi = \varphi + \ln\left(\frac{\cos\tau + r\sin\tau}{1 + \sin\tau\sqrt{1 + r^2}}\right). \tag{3.6}$$

The metric (3.2) is then

$$d\bar{s}^2 = 2GJ\Omega^2\left(-(1 + r^2)d\tau^2 + \frac{dr^2}{1 + r^2} + d\theta^2 + \Lambda^2(d\varphi + rd\tau)^2\right). \tag{3.7}$$

The NHEK geometry has an enhanced $SL(2, \mathbb{R}) \times U(1)$ isometry group [18]. The rotational $U(1)$ isometry is generated by the Killing vector

$$\zeta_0 = -\partial_\varphi. \tag{3.8}$$

Time translations become part of an enhanced $SL(2, \mathbb{R})$ isometry group generated by the Killing vectors

$$\tilde{J}_1 = 2\sin\tau\frac{r}{\sqrt{1 + r^2}}\partial_\tau - 2\cos\tau\sqrt{1 + r^2}\partial_r + \frac{2\sin\tau}{\sqrt{1 + r^2}}\partial_\varphi \tag{3.9}$$

$$\tilde{J}_2 = -2\cos\tau\frac{r}{\sqrt{1 + r^2}}\partial_\tau - 2\sin\tau\sqrt{1 + r^2}\partial_r - \frac{2\cos\tau}{\sqrt{1 + r^2}}\partial_\varphi \tag{3.10}$$

$$\tilde{J}_0 = 2\partial_\tau. \tag{3.11}$$

Note that all of these isometries act within a three-dimensional slice of fixed polar angle θ. The geometry of these slices is a quotient of warped AdS_3 (the AdS_3 analog of the squashed S^3), with the quotient arising from the φ identification [32,33]. Such quotients are (warped) black holes, much as AdS_3 quotients are BTZ (Banados-Teitelboim-Zanelli) black holes [34]. The τ, r plane describes AdS_2, while the φ circle is an S^1 bundle over the AdS_2. At the special value of θ where $\Omega^2 = \sin\theta$, the slice is locally an ordinary AdS_3, and acquires a local $SL(2, \mathbb{R})_R \times SL(2, \mathbb{R})_L$ isometry. At all other values of θ, the $SL(2, \mathbb{R})_L$ is broken to $U(1)$. Near the equator we have a "stretched" AdS_3 quotient (as the S^1 fiber is stretched), while near the poles we have a "squashed" AdS_3 quotient. Properties of these three-dimensional spacetimes in a context relevant to the present one were recently described in [34].

IV. THE ASYMPTOTIC SYMMETRY GROUP

We now turn to the study of excitations around near-horizon extreme Kerr. This requires imposing boundary conditions at the $S^2 \times \mathbb{R}$ boundary $y = 0$. Since we lost the asymptotically flat region in taking the near-horizon limit, this boundary is not flat and it is not *a priori* obvious what boundary conditions we should use. Indeed, different boundary conditions may be relevant in different physical contexts. For every consistent set of boundary conditions there is an associated asymptotic symmetry group (ASG). This is defined as the set of allowed symmetry transformations modulo the set of trivial symmetry transformations

$$\mathrm{ASG} = \frac{\text{Allowed Symmetry Transformations}}{\text{Trivial Symmetry Transformations}}. \tag{4.1}$$

Here "allowed" means that the transformation is consistent with the specified boundary conditions, while "trivial" means that the generator of the transformation vanishes after we have implemented the constraints and reduced it to a boundary integral.

Consistency requires that the generators of the ASG be well defined and not diverge at the boundary. If the boundary conditions are too strong, all interesting excitations are ruled out. If they are too weak, the generators of the ASG are ill-defined. In general, there is a narrow window of consistent boundary conditions. For example, in asymptotically flat space, one usually requires that excitations of the metric fall off like $\frac{1}{r}$ or faster at infinity. The ASG is then simply the Poincaré group. One might try to demand that the metric fall off spatially as $\frac{1}{r^2}$. This would allow only zero energy configurations and hence the theory would be trivial. On the other hand, one might try to demand that it fall off as $\frac{1}{\sqrt{r}}$. Then the energy and other symmetry generators would be in general divergent, and it is unlikely any sense could be made of the theory. So the general idea is to make the falloff weak enough to include the physics of interest, while still maintaining finiteness of the generators.

V. BOUNDARY CONDITIONS

We choose the boundary conditions

$$\begin{pmatrix} h_{\tau\tau} = \mathcal{O}(r^2) & h_{\tau\varphi} = \mathcal{O}(1) & h_{\tau\theta} = \mathcal{O}(\frac{1}{r}) & h_{\tau r} = \mathcal{O}(\frac{1}{r^2}) \\ h_{\varphi\tau} = h_{\tau\varphi} & h_{\varphi\varphi} = O(1) & h_{\varphi\theta} = \mathcal{O}(\frac{1}{r}) & h_{\varphi r} = \mathcal{O}(\frac{1}{r}) \\ h_{\theta\tau} = h_{\tau\theta} & h_{\theta\varphi} = h_{\varphi\theta} & h_{\theta\theta} = \mathcal{O}(\frac{1}{r}) & h_{\theta r} = \mathcal{O}(\frac{1}{r^2}) \\ h_{r\tau} = h_{\tau r} & h_{r\varphi} = h_{\varphi r} & h_{r\theta} = h_{\theta r} & h_{rr} = \mathcal{O}(\frac{1}{r^3}) \end{pmatrix}, \tag{5.1}$$

where $h_{\mu\nu}$ is the deviation of the full metric from the background NHEK metric $\bar{g}$ in (3.7). We note that the allowed deviations $h_{\tau\tau}$ and $h_{\varphi\varphi}$[5] are of the same order as the leading terms in (3.7). In this regard, these boundary conditions differ, for example, from the usual AdS_3 boundary conditions [2], where all deviations are subleading. An analysis with a number of similarities to the present one (with nonsubleading deviations) for the BMS group at $\mathcal{I}^+$ can be found in [35,36]. The most general diffeomorphisms which preserve the boundary conditions (5.1) are of the form

$$\xi = [-r\epsilon'(\varphi) + O(1)]\partial_r + \left[C + O\left(\frac{1}{r^3}\right)\right]\partial_\tau + \left[\epsilon(\varphi) + O\left(\frac{1}{r^2}\right)\right]\partial_\varphi + O\left(\frac{1}{r}\right)\partial_\theta \tag{5.2}$$

where $\epsilon(\varphi)$ is an arbitrary smooth function of the boundary coordinate φ, and C is an arbitrary constant. The subleading terms indicated above can be seen, after computing the generators, to correspond to trivial diffeomorphisms. Therefore the asymptotic symmetry group contains one copy of the conformal group of the circle generated by[6]

$$\zeta_\epsilon = \epsilon(\varphi)\partial_\varphi - r\epsilon'(\varphi)\partial_r. \tag{5.3}$$

This Virasoro algebra here has only a $U(1)$, not an $SL(2,\mathbb{R})$, isometry subgroup.[7] The NHEK metric (3.2) transforms under (5.3) as

$$\delta_\epsilon d\bar{s}^2 = 4JG\Omega^2\bigg(r^2(1-\Lambda^2)\partial_\varphi\epsilon d\tau^2 - \frac{r\partial_\varphi^2\epsilon}{1+r^2}d\varphi dr + \Lambda^2\partial_\varphi\epsilon d\varphi^2 - \frac{\partial_\varphi\epsilon}{(1+r^2)^2}dr^2\bigg). \tag{5.4}$$

Since $\varphi \sim \varphi + 2\pi$ (because $\phi \sim \phi + 2\pi$), it is convenient to define $\epsilon_n(\varphi) = -e^{-in\varphi}$ and $\zeta_n = \zeta(\epsilon_n)$. Under Lie brackets, these symmetry generators obey the Virasoro algebra

$$i[\zeta_m, \zeta_n]_{\mathrm{L.B.}} = (m-n)\zeta_{m+n}. \tag{5.5}$$

Note that ζ_0 generates the $U(1)$ rotational isometry.

The allowed symmetry transformations (5.2) also include τ translations generated by ∂_τ. The conjugate conserved quantity, which we denoted E_R, measures the deviation $\frac{M^2}{G} - J$ of the black hole from extremality. Here we wish to study only the extremal black holes, which entails a restriction to the subspace in which E_R vanishes. This restriction should be compatible with (5.3) because ∂_τ commutes with the Virasoro generators. It can be implemented with an additional boundary condition, given in Sec. VI below, which makes the generator of τ translations trivial.

The reader may wonder how we came up with the boundary conditions (5.1). We began by assuming (a) the

[5]The asymptotic constraints force a linear combination of these, the trace of $h_{\mu\nu}$, to vanish at linear order, as described in Appendix A.

[6]ζ_ϵ is discontinuous at the north and south poles $\theta = (0, \pi)$. This can be regulated by taking for example $\tilde{\zeta}_\epsilon = \frac{r^2\sin\theta}{1+r^2\sin\theta} \times [\epsilon(\varphi)\partial_\varphi - r\epsilon'(\varphi)\partial_r]$. Expanding in $\frac{1}{r}$ we see that $\tilde{\zeta}_\epsilon$ and ζ_ϵ differ by trivial diffeomorphisms, while $\tilde{\zeta}_\epsilon$ is smooth for any finite r.

[7]This suggests that the CFT state dual to the Kerr vacuum is not $SL(2,\mathbb{R})$ invariant.

existence of a nontrivial Virasoro whose zero mode is proportional to ∂_φ in the allowed diffeomorphisms, and (b) the boundary conditions can be linearly described in terms of power law falloff of the individual components of the metric fluctuations. We found only one self-consistent set of boundary conditions with these properties, up to possible further constraints on subleading terms which do not affect the ASG or its central charge. In studies of the Gödel black hole [37] and warped AdS_3 [38], consistent boundary conditions were imposed in which the $SL(2,\mathbb{R})$ isometry is enhanced to a Virasoro algebra, and the $U(1)$ isometry is enhanced to a current algebra. That is quite different than the situation here (as well as in [39]) in which the $SL(2,\mathbb{R})$ becomes trivial and the $U(1)$ is enhanced to a Virasoro and therefore do not meet requirement (a) above. We expect that consistent boundary conditions analogous to those described in [37,38] do exist for Kerr. If so, they are likely relevant to an understanding of the entropy of near-extremal fluctuations [since the $\bar{L}_0$ of the $SL(2,\mathbb{R})$ measures the deviation from extremality] rather than the ground state entropy of extreme Kerr.

VI. GENERATORS

Now we need to construct the surface integrals which generate the diffeomorphisms of (5.3) via Dirac brackets, and see if they are finite. When the deviations h of the metric are not subleading, the charges can have nonlinear corrections, which must be carefully considered. For this purpose the covariant formalism of Barnich, Brandt, and Compère [40,41], based on [42–47] and further developed in [48,49], is the most complete and will be adopted in the following. An example, mathematically quite similar to the present one, are the Gödel black holes analyzed in [37].

The generator of a diffeomorphism ζ is a conserved charge $Q_\zeta[g]$.[8] Under Dirac brackets, the charges associated with asymptotic symmetries obey the same algebra as the symmetries themselves, up to a possible central term. Infinitesimal charge differences between neighboring geometries $g_{\mu\nu}$ and $g_{\mu\nu}+h_{\mu\nu}$ are given by

$$\delta Q_\zeta[g] = \frac{1}{8\pi G}\int_{\partial\Sigma} k_\zeta[h,g] \tag{6.1}$$

where the integral is over the boundary of a spatial slice and

$$\begin{aligned} k_\zeta[h,g] = -\tfrac{1}{4}\epsilon_{\alpha\beta\mu\nu}\Big[&\zeta^\nu D^\mu h - \zeta^\nu D_\sigma h^{\mu\sigma} \\ &+ \zeta_\sigma D^\nu h^{\mu\sigma} + \tfrac{1}{2}hD^\nu\zeta^\mu - h^{\nu\sigma}D_\sigma\zeta^\mu \\ &+ \tfrac{1}{2}h^{\sigma\nu}(D^\mu\zeta_\sigma + D_\sigma\zeta^\mu)\Big]dx^\alpha\wedge dx^\beta. \end{aligned} \tag{6.2}$$

Covariant derivatives and raised indices are computed using $g_{\mu\nu}$. In asymptotically AdS spacetimes, the formula (6.1) for the charge is true even for finite h, and it agrees with the charges obtained in the classic Hamiltonian [2,50,51] or quasilocal [52,53] formalisms. However, in certain cases such as 5d Gödel spacetimes [54,55], nonlinear contributions are important near the boundary, and only infinitesimal h is allowed. In those cases, finite charge differences are computed by integrating δQ over a path in the configuration space,

$$Q_\zeta[g] - Q_\zeta[\bar{g}] = \int_\gamma \delta Q_\zeta[g(\gamma)] \tag{6.3}$$

where γ connects $\bar{g}$ to g and $h(\gamma)$ in (6.1) is taken tangent to the path. Path independence holds provided certain integrability conditions are satisfied [41,49]. We show that these conditions are obeyed around NHEK in Appendix B.

The charges that generate ∂_τ and ζ_ϵ are

$$Q_{\partial_\tau} = \frac{1}{8\pi G}\int_{\partial\Sigma} k_{\partial_\tau}, \qquad Q_{\zeta_\epsilon} = \frac{1}{8\pi G}\int_{\partial\Sigma} k_{\zeta_\epsilon}. \tag{6.4}$$

Choosing $g_{\mu\nu}$ to be the NHEK background, the integrands simplify to

$$\begin{aligned} k_{\partial_\tau} = -\bigg(&\frac{1}{4\Lambda}r\Big[(\Lambda^4+\Lambda^2-2)h_{\varphi\varphi} + \frac{\Lambda^4}{r^2}h_{\tau\tau}\Big] \\ &- \frac{1}{4\Lambda}[r^3\Lambda^4 h_{rr} + 2r^2\Lambda\partial_\theta(\Lambda h_{r\theta}) \\ &+ 2\Lambda^2 r\partial_\tau h_{r\varphi} + 2(\Lambda^2-1)r^2\partial_r h_{\varphi\varphi} + 2\Lambda^4 h_{\tau\varphi} \\ &- \Lambda^2 r(\Lambda^2 - 2 + 2r\partial_r)h_{\theta\theta}]\bigg)d\theta\wedge d\phi + \cdots \end{aligned} \tag{6.5}$$

$$\begin{aligned} k_{\zeta_\epsilon} = \frac{1}{4\Lambda}\Big[&2\Lambda^2\epsilon' r h_{r\varphi} - \epsilon\Lambda^2\Big(\Lambda^2\frac{h_{\tau\tau}}{r^2} + (\Lambda^2+1)h_{\varphi\varphi} \\ &+ 2r\partial_\varphi h_{r\varphi}\Big)\Big]d\theta\wedge d\varphi + \cdots. \end{aligned} \tag{6.6}$$

We have assumed the boundary conditions (5.1) and discarded total φ derivatives. The $+\cdots$ includes terms which vanish for $r\to\infty$ or are not tangent to $\partial\Sigma$, and so do not contribute to the integral. From the boundary conditions (5.1) we see immediately that k_{ζ_ϵ}, and therefore Q_{ζ_ϵ}, are finite around NHEK. For a general background $g_{\mu\nu}$, a straightforward counting of powers of r term by term in (6.2) reveals that Q_{ζ_ϵ} remains finite.

In addition, we must show that Q_{∂_τ}, which measures the deviation from extremality, is well defined. This does not follow immediately from the boundary conditions (5.1).[9] In fact, as we are studying extreme Kerr, we want this charge not only to be finite, but to vanish altogether, i.e. to be trivial. We therefore impose the supplementary boundary

[8] We choose the arbitrary additive constants appearing in [40,41] so that $Q_\zeta[\bar{g}] = 0$ for $\bar{g}$ the NHEK metric.

[9] A similar structure was encountered in [37], which similarly imposes a supplementary boundary condition.

condition

$$E_R \equiv Q_{\partial_\tau}[g] = 0. \tag{6.7}$$

This is equivalent to requiring that the pullback of k_{∂_τ} to the boundary obeys $k_{\partial_\tau}|_{\partial\Sigma} = dX|_{\partial\Sigma}$ for some oneform X globally defined on $\partial\Sigma$. Under the constraint (6.7), only perturbations h which preserve (6.7) and only background metrics g which can be reached from the NHEK geometry via a path of such perturbations are considered. This is presumably a complicated nonlinear submanifold of the geometries allowed by the linear boundary conditions (5.1). It can be shown that the $E_R = 0$ submanifold contains, in particular, finite generalizations of the infinitesimal ζ_ϵ diffeomorphisms acting on the NHEK geometry.[10] These carry nonzero Q_{ζ_ϵ} charges. The inclusion of such spaces is expected because the ζ_ϵ and ∂_τ commute. We do not know if there are other types of spaces with $E_R = 0$. The answer likely depends on the matter content of the theory, about which so far we have assumed only that it does not affect the boundary behavior.

It remains to be seen that, with the supplementary boundary condition (6.7), the transformations ζ_ϵ are still allowed. Formally this follows from the fact that ζ_ϵ and ∂_τ commute, but we must be careful about divergences. It is easy to check directly that the perturbation (5.4), which results from the action of ζ_ϵ on the NHEK geometry, indeed yields a k_{∂_τ} obeying (6.7). For the more general background consistent with (6.7), we use the fact that the generators Q_{ζ_ϵ} are well defined on the bigger space of geometries obeying only (5.1). Therefore, they properly generate the local action of a ζ_ϵ diffeomorphism. This will preserve the local expression $k_{\partial_\tau}|_{\partial\Sigma} = dX|_{\partial\Sigma}$ of k_{∂_τ} as an exact form on $\partial\Sigma$ up to a c number corresponding to a possible central term. The central term is [40]

$$\frac{1}{8\pi G}\int_{\partial\Sigma} k_{\zeta_\epsilon}[\mathcal{L}_\tau \bar{g}, \bar{g}] \tag{6.8}$$

where $\mathcal{L}_\tau$ is the Lie derivative along τ. As there is no possible central term between the generators of Virasoro and τ translations, this must vanish, in agreement with explicit computation. Therefore we can consistently restrict to extremal configurations by imposing (6.7).

VII. CENTRAL CHARGE

The Dirac bracket algebra of the asymptotic symmetry group is computed by varying the charges

$$\{Q_{\zeta_m}, Q_{\zeta_n}\}_{\text{D.B.}} = Q_{[\zeta_m,\zeta_n]} + \frac{1}{8\pi G}\int_{\partial\Sigma} k_{\zeta_m}[\mathcal{L}_{\zeta_n}\bar{g}, \bar{g}]. \tag{7.1}$$

[10]Verifying this by explicit computation is a bit tricky because of subtleties at the north and south pole, and uses the fact that $dk_{\partial_\tau} = 0$ on shell [40]. To make the computation well defined, one must use a regulated form of ζ_ϵ as e.g. given in footnote 6.

For the NHEK geometry the Lie derivative gives

$$\mathcal{L}_{\zeta_n}\bar{g}_{\tau\tau} = 4GJ\Omega^2(1-\Lambda^2)r^2 ine^{-in\varphi} \tag{7.2}$$

$$\mathcal{L}_{\zeta_n}\bar{g}_{r\varphi} = -\frac{2GJ\Omega^2 r}{1+r^2}n^2 e^{-in\varphi} \tag{7.3}$$

$$\mathcal{L}_{\zeta_n}\bar{g}_{\varphi\varphi} = 4GJ\Lambda^2\Omega^2 ine^{-in\varphi} \tag{7.4}$$

$$\mathcal{L}_{\zeta_n}\bar{g}_{rr} = -\frac{4GJ\Omega^2}{(1+r^2)^2} ine^{-in\varphi}. \tag{7.5}$$

It follows that

$$\frac{1}{8\pi G}\int_{\partial\Sigma} k_{\zeta_m}[\mathcal{L}_{\zeta_n}\bar{g}, \bar{g}] = -i(m^3+2m)\delta_{m+n}J. \tag{7.6}$$

Let us now define dimensionless quantum versions of the Q's by

$$\hbar L_n \equiv Q_{\zeta_n} + \frac{3J}{2}\delta_n, \tag{7.7}$$

plus the usual rule of Dirac brackets to commutators as $\{.,.\}_{\text{D.B.}} \to -\frac{i}{\hbar}[.,.]$. The quantum charge algebra is then

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{J}{\hbar}m(m^2-1)\delta_{m+n,0}. \tag{7.8}$$

From this we can read off the central charge for extreme Kerr

$$c_L = \frac{12J}{\hbar}. \tag{7.9}$$

For GRS 1915+105, this gives $c_L = (2\pm 1)\times 10^{79}$, with the uncertainty coming from the uncertainty in the measured mass.

We note that (7.9) does not depend on the details of the boundary conditions (5.1) in that it holds for any boundary conditions as long as the diffeomorphisms (5.3) are allowed.

VIII. TEMPERATURE

In this section we derive the relation $T_L = \frac{1}{2\pi}$ for the generalized temperature of the near-horizon region in units of its inverse radius.

First, we must define the quantum vacuum for extreme Kerr. This problem is subtle because Kerr has no everywhere timelike Killing vector, so in fact, globally, there is no quantum state with all the desired properties of a vacuum. There is an extensive literature on this subject for the generic Kerr black hole, references to which can be found in [56]. Frolov and Thorne [57] define a vacuum by using a Killing vector field which is timelike from the horizon out to the speed of light surface, which is the surface at which an observer must move at the speed of light in order to corotate with the black hole. The Frolov-Thorne vacuum has some pathologies outside of this surface [58], but is well behaved in the near-horizon region [56], where it is an analog of the Hartle-Hawking vacuum for Schwarzschild and is therefore ideal for our purposes.

Construction of the Frolov-Thorne vacuum for generic Kerr begins by expanding the quantum fields in eigenmodes of the asymptotic energy ω and angular momentum m. For example, for a scalar field Φ we may write

$$\Phi = \sum_{\omega,m,l} \phi_{\omega m l} e^{-i\omega\hat{t}+im\hat{\phi}} f_l(r,\theta). \tag{8.1}$$

After tracing over the region inside the horizon, the vacuum is a diagonal density matrix in the energy-angular momentum eigenbasis with a Boltzmann weighting factor

$$e^{-\hbar(\omega-\Omega_H m)/T_H}. \tag{8.2}$$

This reduces to the Hartle-Hawking vacuum in the nonrotating $\Omega_H = 0$ case.

In order to transform this to near-horizon quantities and take the extremal limit (in which $T_H \to 0$) we note that in the near-horizon coordinates

$$e^{-i\omega\hat{t}+im\hat{\phi}} = e^{-(i/\lambda)(2M\omega-m)t+im\phi} = e^{-in_R t+in_L\phi}, \tag{8.3}$$

where

$$n_L \equiv m, \qquad n_R \equiv \tfrac{1}{\lambda}(2M\omega - m) \tag{8.4}$$

are the left and right charges associated to ∂_ϕ and ∂_t in the near-horizon region. In terms of these variables the Boltzmann factor (8.2) is

$$e^{-\hbar(\omega-\Omega_H m)/T_H} = e^{-(n_L/T_L)-(n_R/T_R)}, \tag{8.5}$$

where the dimensionless left and right temperatures are

$$T_L = \frac{r_+ - M}{2\pi(r_+ - a)}, \qquad T_R = \frac{r_+ - M}{2\pi\lambda r_+}. \tag{8.6}$$

In the extremal limit $M^2 \to GJ$ these reduce to

$$T_L = \frac{1}{2\pi}, \qquad T_R = 0. \tag{8.7}$$

The left-movers are then thermally populated with the Boltzmann distribution at temperature $1/2\pi$[11]

$$e^{-2\pi n_L}. \tag{8.8}$$

Note that even though extreme Kerr has zero Hawking temperature, the quantum fields outside the horizon are not in a pure state.

[11]A fast but less rigorous way to derive this result is to note that at every fixed polar angle θ, the geometry is a quotient of warped AdS_3. The temperature for such quotients is the length of the shift determining the quotient divided by $4\pi^2$ [34,59]. This gives $T_L = \frac{1}{2\pi}$ for every θ.

IX. MICROSCOPIC ORIGIN OF THE BEKENSTEIN-HAWKING-KERR ENTROPY

In the previous section we saw that the quantum theory in the Frolov-Thorne vacuum restricted to extreme Kerr has the left-moving temperature

$$T_L = \frac{1}{2\pi}. \tag{9.1}$$

Since the states of quantum gravity on NHEK, with the boundary conditions (5.1), are identified under the holographic duality with those of the left-moving part of the CFT, the CFT dual of the Frolov-Thorne vacuum must also have temperature (9.1). The central charge of the CFT was shown to be

$$c_L = \frac{12J}{\hbar}. \tag{9.2}$$

According to the Cardy formula the entropy for a unitary CFT at large T_L obeys[12]

$$S = \frac{\pi^2}{3} c_L T_L. \tag{9.3}$$

Using (9.1) and (9.2), we find the microscopic entropy for the dual to extreme Kerr

$$S_{\text{micro}} = \frac{2\pi J}{\hbar} = S_{\text{BH}}. \tag{9.4}$$

This exactly reproduces the macroscopic Bekenstein-Hawking entropy (2.11) of the extreme Kerr black hole.

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APPENDIX A: ASYMPTOTIC CONSTRAINTS

In this appendix we work out the asymptotic form of the constraint equations, which relate the leading order fluctuations of the metric. In a Dirac bracket formalism, the constraints, by construction, commute with everything. Therefore the generators of the ASG are ambiguous up to the additions of integrals proportional to the constraints.

[12]A sufficient but not necessary condition for validity of the Cardy formula is $T \gg c$. This condition is not obeyed here, as in many black hole applications [3]. In many such cases the formula is nevertheless valid because of the small gap arising from highly twisted sectors [60]. For example we might expect a twisted sector of order J, which is effectively described by a universal $c_L = 12$ "long string" CFT at temperature $T_L = \frac{J}{2\pi}$. A small gap is generic for black holes [61] so we hope that the same mechanism is operative here.

The constraint equations are $G^0_\mu = 0$. Using the boundary conditions (5.1), linearizing in $h_{\mu\nu}$ and expanding to leading order in $1/r$, we can solve the asymptotic constraint equations as follows.

First consider $G^0_\varphi = 0$. At leading order this is a second order differential equation for $h_{\varphi\varphi}$ in θ, and does not involve the other metric components. The solution which leads to a metric regular at the poles is

$$h_{\varphi\varphi} = \Lambda^2\Omega^2 f(\tau, \varphi) + O(1/r). \tag{A1}$$

Now consider $G^0_0 = 0$. This involves only θ, $h_{\tau\tau}$, $h_{\varphi\varphi}$, and their first and second θ derivatives. Plugging in the solution for $h_{\varphi\varphi}$, all the derivatives drop out and the solution is

$$h_{\tau\tau} = r^2(1-\Lambda^2)\Omega^2 f(\tau, \varphi) + O(r). \tag{A2}$$

Now consider $G^0_\theta = 0$. This is proportional to

$$2\Lambda^2(\Lambda\partial_\theta\Omega - \Omega\partial_\theta\Lambda)h_{r\varphi} - \Lambda^3\Omega\partial_\theta h_{r\varphi} - \Omega\partial_\theta\Lambda\partial_\varphi h_{\varphi\varphi}. \tag{A3}$$

Plugging in the solution above for the θ dependence of $h_{\varphi\varphi}$, we find

$$h_{r\varphi} = -\frac{1}{r}\left(\frac{\Omega^2}{2}\partial_\varphi f(\tau, \varphi) + \frac{16\Omega^2}{\Lambda^2} g(\tau, \varphi)\right) + O\left(\frac{1}{r^2}\right). \tag{A4}$$

Now consider $G^0_r = 0$. This involves θ, $h_{r\varphi}$, $\partial_\theta h_{t\varphi}$, $\partial^2_\theta h_{r\varphi}$ $\partial_\varphi h_{\varphi\varphi}$, and $\partial_\varphi h_{\tau\tau}$. Plugging in the solutions for $h_{\mu\nu}$ from above, the final condition is $g(\tau, \varphi) = 0$. Note that the constraints imply $h \equiv \bar{g}^{\mu\nu}h_{\mu\nu} = 0$.

APPENDIX B: CHARGE INTEGRABILITY

In this appendix we show that to quadratic order around the NHEK background, the charges (6.3) do not depend on the path of integration over metrics, γ. Since $E_R = 0$, only $Q_{\zeta_\epsilon}[g]$ needs to be checked. The integrability condition is

$$\int_{\partial\Sigma}(k_{\zeta_\epsilon}[h, g+\tilde{h}] - k_{\zeta_\epsilon}[\tilde{h}, g+h] - k_{\zeta_\epsilon}[h-\tilde{h}, g]) = 0 \tag{B1}$$

keeping terms up to order $h\tilde{h}$. The integrand is

$$\begin{aligned}
&-\tfrac{1}{8}\epsilon_{\alpha\beta\mu\nu}\Big[\tilde{h}\Big(\zeta^\nu D^\mu h - \zeta^\nu D_\sigma h^{\mu\sigma} + \tfrac{1}{2}h^{\sigma\nu}(D^\mu\zeta_\sigma + D_\sigma\zeta^\mu)\Big) \\
&\quad + \zeta^\nu h^{\lambda\mu}D_\lambda\tilde{h} - \zeta^\nu(2D_\sigma\tilde{h}^\mu_\lambda - D^\mu\tilde{h}_{\lambda\sigma})h^{\lambda\sigma} \\
&\quad + \zeta^\sigma h^{\lambda\nu}D_\sigma\tilde{h}^\mu_\lambda - (h_{\sigma\lambda}\tilde{h}^{\nu\lambda}D^\mu\zeta^\sigma + h^\nu_\sigma\tilde{h}^{\mu\lambda}D_\lambda\zeta^\sigma) \\
&\quad - (h \leftrightarrow \tilde{h})\Big]dx^\alpha \wedge dx^\beta.
\end{aligned} \tag{B2}$$

Using the boundary conditions (5.1) and the constraints $h = \tilde{h} = 0$ derived in Appendix A, the component tangent to $\partial\Sigma$ vanishes on the NHEK background $\bar{g}_{\mu\nu}$ for $\zeta = \zeta_\epsilon$.

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