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Lösung der Wellengleichung für das Elektron
in Gegenwart einer ebenen Lichtwelle.

$$A = a e^{i\nu(t - \frac{nx}{c})} + \bar{a} e^{-i\nu(t - \frac{nx}{c})} \quad (1)$$

Wellengl. $\left\{ \frac{E}{c} + \beta_1(\alpha p) + \beta_3 mc + \frac{e}{c} \beta_1(\alpha A) \right\} \psi = 0 \quad (2)$

$$\varphi \left\{ \frac{E}{c} + \beta_1(\alpha p) + \beta_3 mc + \frac{e}{c} \beta_1(\alpha A) \right\} = 0 \quad (2a)$$

$$\sigma_1 \sigma_2 = i \sigma_3 \text{ etc} \quad (3)$$

$$\beta_1 \beta_2 = i \beta_3 \text{ etc} \quad (3a)$$

Kontinuitätsgl.

$$\frac{\partial(\varphi \psi)}{\partial t} + \text{div}(-c \varphi \beta_1 \alpha \psi) = 0 \quad (4)$$

in Übereinstimmung mit

$$\rho = -e \varphi \psi, \quad J = ec \varphi \beta_1 \alpha \psi \quad (5)$$

$$\psi_0 = u e^{\frac{i}{\hbar}[-Et + (px)]}, \quad \varphi_0 = v e^{-\frac{i}{\hbar}[-Et + (px)]} \quad (6)$$

$$-\frac{E^2}{c^2} + p^2 + m^2 c^2 = 0 \quad (7)$$

$$\text{oder } E = \pm c \sqrt{p^2 + m^2 c^2}$$

$$\left. \begin{aligned} \psi &= (1 + f e^{i\nu(\cdot)} + \bar{f} e^{-i\nu(\cdot)}) \psi_0 \\ \varphi &= \varphi_0 (1 + g e^{i\nu(\cdot)} + \bar{g} e^{-i\nu(\cdot)}) \end{aligned} \right\} \quad (8)$$

$$\left. \begin{aligned} f &= -\frac{e}{2\hbar v(\frac{E}{c} - (np))} [2(a\rho) + \hbar(\sim\gamma) + i\hbar f_1(\sim E)] \\ \tilde{f} &= \frac{e}{2\hbar v(\frac{E}{c} - (np))} [2(\tilde{a}\rho) + \hbar(\sim\tilde{\gamma}) + i\hbar f_1(\sim\tilde{E})] \\ g &= \frac{e}{2\hbar v(\frac{E}{c} - (np))} [2(a\rho) + \hbar(\sim\gamma) - i\hbar f_1(\sim E)] \\ \tilde{g} &= -\frac{e}{2\hbar v(\frac{E}{c} - (np))} [2(\tilde{a}\rho) + \hbar(\sim\tilde{\gamma}) - i\hbar f_1(\sim\tilde{E})] \end{aligned} \right\} \quad (9)$$

$$\left. \begin{aligned} Q &= \varepsilon e^{i v(t - \frac{nx}{c})} + \tilde{\varepsilon} e^{-i v(t - \frac{nx}{c})} \\ H &= \gamma e^{i v(t - \frac{nx}{c})} + \tilde{\gamma} e^{-i v(t - \frac{nx}{c})} \end{aligned} \right\} \quad (10)$$

Allgemeine Lösung $\int \psi(p) dp, \int \varphi(p) dp.$

Wahrscheinlichkeitsdichte $\int \varphi(p) \psi(p') dp dp'$

$$\begin{aligned} \varphi(p) \psi(p') &= \varphi_0(p) \psi_0(p') + v(p) \left\{ [f(p') + g(p)] e^{\frac{i}{\hbar} [-(E'-E-\hbar v)t + (p'-p - \frac{\hbar v}{c})x]} \right. \\ &\quad \left. + [\tilde{f}(p') + \tilde{g}(p)] e^{\frac{i}{\hbar} [-(E'-E+\hbar v)t + (p'-p + \frac{\hbar v}{c})x]} \right\} u(p') \end{aligned}$$

Nach Weglassung von $\varphi_0 \psi_0$

$$\begin{aligned} \iint dp dp' &\left\{ v(p) [f(p') + g(p)] u(p') e^{\frac{i}{\hbar} [-(E'-E-\hbar v)t + (p'-p - \frac{\hbar v}{c})x]} \right. \\ &\quad \left. + v(p') [\tilde{f}(p) + \tilde{g}(p')] u(p) e^{\frac{i}{\hbar} [-(E'-E+\hbar v)t + (p'-p + \frac{\hbar v}{c})x]} \right\} \end{aligned}$$

Elektrische Dichte

$$\begin{aligned} P(p) &= -e \int dp' \left\{ v(p) [f(p') + g(p)] u(p') e^{\frac{i}{\hbar} [-(E'-E-\hbar v)t + (p'-p - \frac{\hbar v}{c})x]} \right. \\ &\quad \left. + v(p') [\tilde{f}(p) + \tilde{g}(p')] u(p) e^{\frac{i}{\hbar} [-(E'-E+\hbar v)t + (p'-p + \frac{\hbar v}{c})x]} \right\} \end{aligned} \quad (11)$$

Stromausdruck

$$J(p) = ee \int dp' \left\{ v(p) [\beta_1 \alpha f(p') + g(p) \beta_1 \alpha] u(p') e^{-\frac{i}{\hbar} [\dots]} \right. \\ \left. + v(p') [\beta_1 \alpha \tilde{f}(p) + \tilde{g}(p') \beta_1 \alpha] u(p) e^{\frac{i}{\hbar} [\dots]} \right\} \quad (12)$$

$$A'(p) = \frac{ee}{c^2} \int dp' \left\{ e^{\frac{i}{\hbar} [E + \hbar \nu - E'] (t - \frac{z}{c})} \int dx e^{-\frac{i}{\hbar} [p - p' + n \frac{\hbar \nu}{c} - n' \frac{E + \hbar \nu - E'}{c}] x} \right. \\ \left. + e^{-\frac{i}{\hbar} (E + \hbar \nu - E') (t - \frac{z}{c})} \int dx e^{\frac{i}{\hbar} [p - p' + n \frac{\hbar \nu}{c} - n' \frac{E + \hbar \nu - E'}{c}] x} \right. \\ \left. v(p) [\beta_1 \alpha f(p') + g(p) \beta_1 \alpha] u(p') \right. \\ \left. + v(p') [\beta_1 \alpha \tilde{f}(p) + \tilde{g}(p') \beta_1 \alpha] u(p) \right\}$$

wobei $\int dx = \int dx_1 \int dx_2 \int dx_3$

$$A' = \frac{(2\pi\hbar)^3 e}{2} \left\{ v(p) [\beta_1 \alpha f(p') + g(p) \beta_1 \alpha] u(p') e^{i\nu'(t - \frac{z}{c})} \right. \\ \left. + v(p') [\beta_1 \alpha \tilde{f}(p) + \tilde{g}(p') \beta_1 \alpha] u(p) e^{-i\nu'(t - \frac{z}{c})} \right\} \quad (13)$$

wobei
$$\left. \begin{aligned} E + \hbar \nu &= E' + \hbar \nu' \\ p + n \frac{\hbar \nu}{c} &= p' + n' \frac{\hbar \nu'}{c} \end{aligned} \right\} \quad (14)$$

$$H' = -i \frac{\nu'}{c} [n' A'] \quad (15)$$

$$p = 0, \quad E = mc^2 \quad (16)$$

$$\therefore g(p) = \frac{e}{2\nu mc} [(\alpha \gamma) - i \beta_1 (\alpha \mathcal{E})]$$

$$\text{+ da } (\alpha \gamma) \alpha = \gamma + i [\alpha \times \gamma]$$

$$\therefore g(p) \beta_1 \alpha = \frac{e}{2\nu mc} \left\{ \beta_1 (\gamma + i [\alpha \times \gamma]) - i (\mathcal{E} + i [\alpha \mathcal{E}]) \right\}$$

Ersetzung von $\beta_1 + \beta_3 \alpha$ durch $1 + \alpha$

$$\left. \begin{aligned} v(p)(1 + \beta_3) &= 0 \\ \left[\frac{E'}{c} + \beta_1 (\sigma p') + \beta_3 mc \right] u(p') &= 0 \end{aligned} \right\} \quad (16)$$

$$\therefore \left. \begin{aligned} v(p)(\beta_1 + i\beta_2) &= 0 \\ \left(\frac{E'}{c} \beta_1 + (\sigma p') - i\beta_2 mc \right) u(p') &= 0 \end{aligned} \right\} \quad (16')$$

$$\therefore v(p)\beta_1 u(p') = - \frac{v(p)(\sigma p')u(p')}{\frac{E'}{c} + mc}$$

$$\text{Setze } \left. \begin{aligned} v(p) \sim u(p') &= S & v(p') \sim u(p) &= \bar{S} \\ v(p) u(p') &= d & v(p') u(p) &= \bar{d} \end{aligned} \right\} \quad (17)$$

$$\left. \begin{aligned} \frac{E'}{c} + mc &= \frac{\hbar}{c} (v - v' + \frac{2mc^2}{\hbar}) \\ p' &= \frac{\hbar}{c} (nv - n'v') \end{aligned} \right\} \quad (18)$$

$$\therefore v(p)\beta_1 u(p') = \frac{-S(nv - n'v')}{v - v' + \frac{2mc^2}{\hbar}} \quad (19)$$

Aus (16)

$$v(p)(\beta_1 \alpha + i\beta_2 \alpha) = 0$$

$$\left\{ \frac{E'}{c} \beta_1 \alpha + \alpha (\sigma p') - i\beta_2 \alpha mc \right\} u(p') = 0$$

$$\begin{aligned} \therefore v(p)\beta_1 \alpha u(p') &= - \frac{v(p) \alpha (\sigma p') u(p')}{\frac{E'}{c} + mc} = - \frac{p' v(p) u(p') + i[p', v \alpha u]}{\frac{E'}{c} + mc} \\ &= \frac{-(nv - n'v')d + i[(nv - n'v'), S]}{v - v' + \frac{2mc^2}{\hbar}} \end{aligned} \quad (20)$$

$$\therefore v(p) \gamma(p) \rho, \sim u(p')$$

$$= \frac{e}{2vmc} \left\{ \frac{-(S, (nv - n'v')) \gamma + i [(nv - n'v') d + i [(nv - n'v'), S], \gamma]}{v - v' + \frac{2mc^2}{h}} - i (d \varepsilon + i [S \varepsilon]) \right\}$$

$$\therefore v(p) \rho, \sim f(p') u(p') = - \frac{e v(p) \rho, \sim}{2h v (\frac{E'}{c} - (np'))} [2(a \bar{\rho}') + h(\alpha \gamma) + i h \rho, (\alpha \varepsilon)] u(p')$$

Ans (14)

$$\frac{E'}{c} - (np') = mc \frac{v'}{v} \quad (21)$$

From

$$v(p) \rho, \sim (\alpha \gamma) u(p') = v(p) \rho, [\gamma + i [\gamma \alpha]] u(p')$$

$$= \frac{-(S, nv - n'v') \gamma + i [\gamma \alpha ((nv - n'v') d + i [(nv - n'v'), S])]}{v - v' + \frac{2mc^2}{h}}$$

$$\& v(p) \sim (\alpha \varepsilon) u(p') = v(p) (\varepsilon + i [\varepsilon \alpha]) u(p') = \varepsilon d + i [\varepsilon S]$$

also

$$v(p) \rho, \sim f(p') u(p') = \frac{e}{2mc v'} \left\{ \frac{\frac{2}{c} (a, nv - n'v') ((nv - n'v') d + i [(nv - n'v'), S])}{v - v' + \frac{2mc^2}{h}} + \frac{(S, nv - n'v') \gamma + i [\gamma, ((nv - n'v') d + i [(nv - n'v'), S])]}{v - v' + \frac{2mc^2}{h}} - i \varepsilon d + [\varepsilon S] \right\}$$

$$\therefore v(p) (\rho, \sim f(p') + \gamma(p) \rho, \sim) u(p') = \frac{ie}{2mc} d \left\{ - \frac{\frac{1}{v} [(nv - n'v') \gamma]}{v - v' + \frac{2mc^2}{h}} - \frac{\varepsilon}{v} + \right. \\ \left. - \frac{\frac{1}{v'} \frac{2i}{c} (a, nv - n'v') (nv - n'v') + \frac{1}{v'} [\gamma, nv - n'v']}{v - v' + \frac{2mc^2}{h}} - \frac{\varepsilon}{v'} \right\} + \\ + \frac{e}{2mc} \left\{ \frac{1}{v} \left(- \frac{(S, nv - n'v') \gamma - [[nv - n'v', S], \gamma]}{v - v' + \frac{2mc^2}{h}} + [S \varepsilon] \right) + \right.$$

$$\begin{aligned}
& + \frac{1}{v'} \left\{ \frac{2e}{c} (a, n\nu - n'\nu') \frac{[n\nu - n'\nu', S] + (S, n\nu - n'\nu') \gamma - [\gamma, [n\nu - n'\nu', S]]}{\nu - \nu' + \frac{2mc^2}{h}} + [\mathcal{E} S] \right\} = \\
& = \frac{eid}{2mc(\nu - \nu' + \frac{2mc^2}{h})} \left\{ \left(\frac{1}{\nu} + \frac{1}{\nu'}\right) [\gamma, n\nu - n'\nu'] - \frac{2(\mathcal{E}n')}{\nu} (n\nu - n'\nu') - \mathcal{E} \left(\frac{1}{\nu} + \frac{1}{\nu'}\right) \cdot \left(\right. \right. \\
& \left. \left. (\nu - \nu' + \frac{2mc^2}{h}) \right\} + \frac{e}{2mc(\nu - \nu' + \frac{2mc^2}{h})} \left\{ \left(\frac{1}{\nu'} - \frac{1}{\nu}\right) (S, n\nu - n'\nu') \gamma + \left(\frac{1}{\nu} + \frac{1}{\nu'}\right) ([n\nu - n'\nu', \gamma]) \right. \\
& \left. + \frac{\mathcal{E}}{\nu} (\mathcal{E}n') [n\nu - n'\nu', S] + \left(\frac{1}{\nu} - \frac{1}{\nu'}\right) [S \mathcal{E}] (\nu - \nu' + \frac{2mc^2}{h}) \right\}
\end{aligned}$$

wo $(na) = 0$ $\mathcal{E} = -\frac{iv}{c} a$ (22)

$$\gamma = [n \mathcal{E}]$$

$$\therefore [n, n\nu - n'\nu'] = \nu' (n' \mathcal{E}) n + (\nu - \nu' (nn')) \mathcal{E}$$

mit Hilfe von (14)

$$\begin{aligned}
& = \frac{eid}{2mc(\nu - \nu' + \frac{2mc^2}{h})} \left\{ \frac{1}{\nu} (n' \mathcal{E}) ((\nu' - \nu) n + 2\nu' n') - \nu' \left(\frac{1}{\nu} + \frac{1}{\nu'}\right)^2 \frac{mc^2}{h} \mathcal{E} \right\} \\
& + \frac{e}{2mc(\nu - \nu' + \frac{mc^2}{h})} \left[\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) \left\{ (S, n\nu - n'\nu') [n \mathcal{E}] + \frac{2\nu'}{\nu - \nu'} (\mathcal{E}n') [n\nu - n'\nu', S] \right. \right. \\
& \left. \left. - [S, \mathcal{E}] (\nu - \nu' + \frac{2mc^2}{h}) \right\} - \left(\frac{1}{\nu} + \frac{1}{\nu'}\right) \left((n [\mathcal{E} S]) (n\nu - n'\nu') \right. \right. \\
& \left. \left. + \nu' (n' [n \mathcal{E}]) S \right) \right]
\end{aligned}$$

Wenn wir die von Dirac für ρ und σ gegebenen matrixausdrücke benutzen, so sind alle drei Komponenten von ρ , σ ebenso wie ρ_3 hermitesche Matrizen. Wir können deshalb annehmen, dass $v(\rho) \neq u(\rho)$ für alle Werte von ρ konjugiert komplex sind. Hieraus folgt auch, dass wir $\varphi(\rho) \neq \psi(\rho)$ als konjugiert komplex annehmen können, was bei den Ausdrücken (8) in der Tat erfüllt ist.

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$\therefore v(p) [\beta, \alpha f(p) + q(p) \beta, \alpha] u(p') + v(p') [\beta, \alpha \tilde{f}(p) + \tilde{q}(p') \beta, \alpha] u(p)$
 konjugiert komplex sind, so dass der Ausdruck für das Vektor-
 potential reell ist. $H' = -\frac{1}{c} v' [n' A']$

$$H' = \frac{(2\pi\hbar)^3 e^2 v'}{2mc^2(v-v' + \frac{2mc^2}{\hbar})} \left\{ d \left(\frac{1}{v} (n' \varepsilon) (v' - v) [n' n] - v' \left(\frac{1}{v} + \frac{1}{v'} \right)^2 \frac{mc^2}{\hbar} [n' \varepsilon] \right) - \right.$$

$$- i \left(\frac{1}{v} - \frac{1}{v'} \right) \left[(S, n v - n' v') \left((n' \varepsilon) n - (n' n) \varepsilon \right) + (v - v' + \frac{2mc^2}{\hbar}) \left((S n') \varepsilon - (n' \varepsilon) S \right) \right]$$

$$+ \frac{2}{v} (\varepsilon n') \left((n' S) (n v - n' v') + (v' - (n n') v) S \right) -$$

$$\left. - \left(\frac{1}{v} + \frac{1}{v'} \right) \left((n [\varepsilon S]) v [n' n] + v' (n' [n \varepsilon]) [n' S] \right) \right\} e^{i v' (t - \frac{z}{c})} +$$

+ konjugiert Komplexes Glied (23)

Für $p=0$, $(1+\beta_3) u = 0$ $v(1+\beta_3) = 0$

$$1+\beta_3 = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\therefore u_1 = u_2 = v_1 = v_2 = 0$, so dass u_3, u_4 & v_3, v_4 die allein
 übrigbleibenden Größen sind. Da sich der Fall eines beliebigen
 p durch eine Lorentztransformation immer auf den Fall $p=0$
 zurückführen lässt, so folgt, dass $u(p)$ & $v(p)$ immer
 nur je zwei unabhängigen Größen enthalten. Insbesondere
 können wir im allgemeinen u, u_1 durch u_3, u_4 ausdrücken
 und ebenso v, v_2 durch v_3, v_4 . Anstatt der konjugiert-
 komplexen Grösse u_3, v_3 & u_4, v_4 können wir folgende
 reelle unabhängige Grösse $a, u, \bar{a}, \bar{u}$ in der in u_3

$$\left. \begin{aligned} u_3 &= a_1 e^{i\delta_1}, & u_4 &= a_2 e^{i\delta_2} \\ v_3 &= a_1 e^{-i\delta_1}, & v_4 &= a_2 e^{-i\delta_2} \end{aligned} \right\} \quad (24)$$

Das Vorhandensein der zwei unabhängig wählbaren entspricht den zwei Einstellmöglichkeiten des Elektrons in einem Magnetfeld. Gehen wir auf eine nähere Untersuchung der Eigenfunktionen bei Anwesenheit eines Magnetfelds ein, so gehen wir von der doppelten Einstellmöglichkeit dadurch Rechnung tragen, dass wir die zu einem p gehörige Lösung auf 2 normieren. Dies bedeutet

$$\int \psi_0(p') \psi_0(p'') dx = 2 \delta(p' - p'')$$

wo $\delta(p' - p'')$ die von Dirac eingeführte singuläre Funktion bezeichnet. Durch Einsetzung des Ausdruck (6) & Vergleich mit dem Fourierschen Integraltheorem

$$v(p) u(p) = \frac{2}{(2\pi\hbar)^3} \quad (25)$$

Wir können leicht diese Bedingung in eine Beziehung für die Grösse a_1 & a_2 verwandeln

$$\left\{ \frac{E}{c} + \beta_1(\sigma p) + \beta_3 mc \right\} u(p) = 0$$

Multiplikation mit $v(p) \beta_3$

$$v(p) \left\{ \beta_3 \frac{E}{c} + i\beta_2(\sigma p) + mc \right\} u(p) = 0$$

ähnlich aus der adjungierten Gl.

$$v(p) \left\{ \beta_3 \frac{E}{c} - i\beta_2(\sigma p) + mc \right\} u(p) = 0$$

$$\therefore v(p) \beta_3 u(p) = - \frac{mc^2}{E} v(p) u(p)$$

$$\text{od. } \frac{1}{2} v(p) (1 - \beta_3) u(p) = \frac{1}{2} \left(1 + \frac{mc^2}{E} \right) v(p) u(p)$$

auch

Die Grösse linker Hand ist aber eben $v_3 u_3 + v_4 u_4 = a_1^2 + a_2^2$, 9.

$$\therefore a_1^2 + a_2^2 = \frac{1}{(2\pi\hbar)^3} \left(1 + \frac{mc^2}{E}\right) \quad (26)$$

In dem Fall $p=0$ kann man leicht zeigen, dass die Anwesenheit eines magnetischen Felds eine Aufhebung der Entartung bewirkt, die wenn das Feld die Richtung von x_3 hat und wenn angenommen wird, dass σ_3 ebenso wie β_3 eine Diagonalmatrix ist, sich so ausdrücken lässt, dass nunmehr nur solche Lösungen existieren bei denen entweder u_3, v_3 od. u_4, v_4 allein von Null verschieden sind. Wir wollen als Anfangszustand ein solches in der x_3 -Richtung "polarisiertes" Elektron annehmen, wo also entweder

$$u_3 v_3 = \frac{1}{(2\pi\hbar)^3}, \quad u_4 = v_4 = 0$$

$$\text{od. } u_4 v_4 = \frac{1}{(2\pi\hbar)^3}, \quad u_3 = v_3 = 0$$

Auch bei beliebigen p führt die Anwesenheit eines Magnetfelds bestimmte Beziehungen mit sich, auf die wir jedoch hier nicht eingehen werden. Wie man aber leicht sieht, * bekommt man bei der erwähnten Polarisationsrichtung der Elektronen u. der getroffenen Matrixwahl nur dann bei allen Winkeln (insbesondere $n'=n$) eine von dem Zeichen der Polarisation unabhängige Intensität der Streustrahlung, wenn die beiden Grösse $v_3(p') u_3(p')$ und $v_4(p') u_4(p')$ die den Endzustand charakterisieren einander gleich sind, d. h. wenn

$$a_1'^2 = a_2'^2 = \frac{1}{2(2\pi\hbar)^3} \left(1 + \frac{mc^2}{E'}\right) = \mathcal{J} \quad (27)$$

wie dies ja jedenfalls bei $p'=0$ der Fall sein würde.

Wenn wir die Grösse H'^2 berechnen, so bekommen wir wie ersichtlich einen linearen Ausdruck in den Grössen d, s und $\bar{d}, \bar{s}$, denn wir sodann nach den Phasen von u & v in Anfangs- und Endzustand mitteln müssen. Wir können nun leicht mit Hilfe der Diracschen Matrixdarstellung der Grössen $\sigma_1, \sigma_2, \sigma_3$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (28)$$

die Mittelwerte von $d\bar{s}_1, s_1\bar{s}_2$ etc. ermitteln. Nehmen wir ersten an, dass $v_3(p)u_3(p)=1$, $v_4(p)u_4(p)=0$, so folgt aus der Definition (7) der Grössen s & $\bar{s}$, d & $\bar{d}$

$$\left. \begin{aligned} s_1 &= v_3(p)u_4(p'), & s_2 &= -i v_3(p)u_4(p'), & s_3 &= v_3(p)u_3(p') \\ \bar{s}_1 &= v_4(p')u_3(p), & \bar{s}_2 &= i v_4(p')u_3(p), & \bar{s}_3 &= v_3(p')u_3(p) \end{aligned} \right\} \quad (29)$$

$$d = v_3(p)u_3(p'), \quad \bar{d} = v_3(p')u_3(p)$$

Es folgt

$$\begin{aligned} (s\bar{s}) &= v_3(p)u_3(p) (2v_4(p')u_4(p') + v_3(p')u_3(p')) \\ &= \frac{3}{2(2\pi\hbar)^6} \left(1 + \frac{mc^2}{E'}\right) = 3\mathcal{V} \end{aligned} \quad (30)$$

Weiter

$$\begin{aligned} [s\bar{s}]_1 &= [s\bar{s}]_2 = 0, & [s\bar{s}]_3 &= s_1\bar{s}_2 - s_2\bar{s}_1 = 2i v_3(p)u_3(p) v_4(p')u_4(p') \\ &= 2i \frac{1}{2(2\pi\hbar)^6} \left(1 + \frac{mc^2}{E'}\right) = 2i\mathcal{V} \end{aligned} \quad (31)$$

wo der Strich über die Grössen $[s\bar{s}]$ den Mittelwert nach den Phasen andeutet. Wenn wir mit ℓ einen Einheitsvektor in der Richtung des polarisierenden Magnetfeldes bezeichnen (in dem eben betrachteten Fall die x_3 -Richtung) so gilt also allgemein

$$\overline{[S\bar{S}]} = \frac{2i}{2(2\pi\hbar)^6} \left(1 + \frac{mc^2}{E'}\right) l \quad (32)$$

wenn ~~W~~ wir den zu dem Eigenwert $+1$ von (σ_l) gehörige Lösung als Anfangszustand annehmen, und ebenso bei der anderen Lösung wo $(\sigma_l) = -1$

$$\overline{[S\bar{S}]} = -\frac{2i}{2(2\pi\hbar)^6} \left(1 + \frac{mc^2}{E'}\right) l \quad (32a)$$

also allgemein $\overline{[S\bar{S}]} = 2i(\sigma_l)l$.

Seien A & B 2 beliebige Vektoren, so bekommen wir ebenso

$$\overline{(AS)(B\bar{S})} = ((AB) + i(A_1B_2 - A_2B_1))\gamma$$

oder allgemein

$$\overline{(AS)(B\bar{S})} = \{ (AB) + i(\sigma_l)([AB]) \} \gamma \quad (33)$$

$$\& [AS][B\bar{S}] = \{ 2(AB) + i(\sigma_l)([AB]) \} \gamma$$

Ferner

$$\overline{(AS)d} = A_3\gamma = (\sigma_l)(A_l)\gamma \quad \& \quad \overline{d(B\bar{S})} = (\sigma_l)(B_l)\gamma \quad (34)$$

& also auch

$$\overline{s d} = \overline{d \bar{s}} = \gamma l(\sigma_l) \quad (35)$$

$$\& \text{endlich: } d\bar{d} = \gamma \quad (36)$$

Mit Hilfe von diesen Rechenregeln können wir nun den Ausdruck H'^2 ausrechnen. Wir müssen dabei ~~den~~ in (23) ausgeschriebenem Ausdruck mit seinem konj. kompl. Wert multiplizieren, den sich ergibt in dem wir erstens i durch $-i$, zweitens σ durch σ^* & drittens S & d durch $\bar{S}$ & $\bar{d}$ ersetzen: Wenn es sich um linear polarisierte

nicht handelt, wie wir zuerst annehmen wollen, können wir E als reell annehmen, so dass $\bar{E} = E$ wird. Es lassen sich dann nach (35) alle Produktglieder von d mit $\bar{S}$ & $\bar{d}$ mit S weg. Dies bedeutet, dass der Teil der S -glieder strahlung, welche mit der d -glied strahlung kohärent ist senkrecht zu der letzteren polarisiert ist. Man sieht nun weiter aus den Ausdrücken (27), dass der Teil der strahlung welche mit der komponent von S nach der Richtung l polarisiert ist, mit der d -strahlung kohärent ist, während der Teil der von der zu l senkrechten Komponente von S abhängt mit der übrigen strahlung inkohärent ist. Es ist deshalb zweckmässig S in zwei Komponenten S_1 und S_2 zu zerlegen, von denen erstere mit l parallel ist während letztere zu dieser Richtung senkrecht steht. Wir setzen also

$$S = S_1 + S_2$$

wo $S_1 = (S.l)l$, $S_2 = S - (S.l)l = [l[S.l]]$

& entsprechend für $\bar{S}$.

Da der Ausdruck (23) in S & $\bar{S}$ linear ist, zerfällt er einfach in zwei Ausdrücken wo S bzw. durch S_1 & S_2 ersetzt ist. Um die entsprechenden Intensitäten berechnen zu können, müssen wir den Formeln (30) bis (36) entsprechende Rechenregeln für S_1 , $\bar{S}_1$, & S_2 , $\bar{S}_2$ aufstellen. Es folgt unmittelbar

$$(S_1, \bar{S}_1) = (S.l)(\bar{S}.l) = \gamma \quad (38)$$

$$(S_2, \bar{S}_2) = 2\gamma$$

Ferner $\overline{[S, \bar{S}_1]} = 0$ $\overline{[S_2, \bar{S}_2]} = \overline{[S, \bar{S}]} = 2i\hbar L(\sigma_L)$ (39)

$\overline{(A S_1)(B \bar{S}_1)} = (A L)(B L)\hbar$, $\overline{(A S_2)(B \bar{S}_2)} = \overline{(A S)(B \bar{S})} - \overline{(A S_1)(B \bar{S}_1)}$

$= \{ (AB) - (A L)(B L) + i\hbar L(\sigma_L)(L[AB]) \}\hbar$ (40)

$[A S_1][B \bar{S}_1] = \{ (AB) - (A L)(B L) \}\hbar$

$[A S_2][B \bar{S}_2] = \{ (AB) + (A L)(B L) + i\hbar L(\sigma_L)(L[AB]) \}\hbar$

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$$\left\{ d \left(\frac{1}{\gamma} (n' \varepsilon) (\gamma' - \gamma) [n' n] - \gamma' \left(\frac{1}{\gamma} + \frac{1}{\gamma'} \right)^2 \frac{mc^2}{h} [n' \varepsilon] \right) \right. \\ \left. - i \left(\left(\frac{1}{\gamma'} - \frac{1}{\gamma} \right) \left[(S, n\gamma - n'\gamma') \left((n' \varepsilon) n - (nn') \varepsilon \right) + (\gamma - \gamma' + \frac{2mc^2}{h}) \left((sn') \varepsilon - (n' \varepsilon) s \right) \right] \right. \right. \\ \left. \left. + \frac{2}{\gamma} (\varepsilon n') \left((n' s) (n\gamma - n'\gamma') + (\gamma' - (nn') \gamma) s \right) - \right. \right. \\ \left. \left. - \left(\frac{1}{\gamma'} + \frac{1}{\gamma} \right) \left((n [\varepsilon s]) \gamma [n' n] + \gamma' (n' [n \varepsilon]) [n' s] \right) \right) \right\}$$

$$\frac{1}{\gamma} (\gamma' - \gamma) = -\frac{\beta}{1+\beta}$$

$$\gamma' \left(\frac{1}{\gamma} + \frac{1}{\gamma'} \right)^2 \frac{mc^2}{h} = \gamma' \frac{1}{\gamma^2} (2+\beta)^2 \frac{\gamma}{\alpha} = \frac{\gamma'}{\gamma} (2+\beta)^2 \frac{1}{\alpha} = \frac{(2+\beta)^2}{(1+\beta)\alpha}$$

$$d \left(\frac{1}{\gamma} (n' \varepsilon) (\gamma' - \gamma) [n' n] - \gamma' \left(\frac{1}{\gamma} + \frac{1}{\gamma'} \right)^2 \frac{mc^2}{h} [n' \varepsilon] \right)$$

$$= d \left((n' \varepsilon) \frac{\beta}{1+\beta} [n' n] - \frac{(2+\beta)^2}{\alpha(1+\beta)} [n' \varepsilon] \right)$$

$$= \frac{d}{1+\beta} \left\{ [n', n \cdot \beta (n' \varepsilon)] - [n', \frac{\varepsilon (2+\beta)^2}{\alpha}] \right\} = \frac{d}{1+\beta} \left\{ [n', n \cdot \beta (n' \varepsilon) - \frac{\varepsilon (2+\beta)^2}{\alpha}] \right\}$$

$$= \frac{d(2+\beta)^2}{\alpha(1+\beta)} \left\{ [n', \frac{\alpha \beta n}{(2+\beta)^2} (n' \varepsilon) - \varepsilon] \right\}$$

$$= - \frac{d \left\{ 2 + \alpha(1 - (nn')) \right\}^2}{\alpha \left\{ 1 + \alpha(1 - (nn')) \right\}} \left[n', \varepsilon - n \cdot \frac{\alpha^2 (1 - (nn'))}{\left\{ 2 + \alpha(1 - (nn')) \right\}^2} (n' \varepsilon) \right] = dA$$

$$S = v(p) \sim u(p')$$

$$d = v(p) u(p')$$

$$\bar{S} = v(p') \sim u(p)$$

$$\bar{d} = v(p') u(p)$$

$$\beta = \alpha(1 - (nn'))$$

$$\left(\frac{1}{\gamma'} - \frac{1}{\gamma} \right) \left\{ (S, n\gamma - n'\gamma') \left((n' \varepsilon) n - (nn') \varepsilon \right) + (\gamma - \gamma' + \frac{2mc^2}{h}) \left((sn') \varepsilon - (n' \varepsilon) s \right) \right\}$$

$$= \frac{\beta}{\gamma} \left\{ \gamma (S, n - n' \frac{1}{1+\beta}) (n' \varepsilon) n + \left(-\gamma (S, n - \frac{n'}{1+\beta}) (nn') + k(sn') \right) \varepsilon - k(n' \varepsilon) s \right\}$$

$$+ \frac{2}{\gamma} (\varepsilon n') \left((n' s) (n\gamma - n'\gamma') + (\gamma' - (nn') \gamma) s \right)$$

$$= \frac{2}{\gamma} (n' \varepsilon) \left\{ \gamma (n' s) \left(n - \frac{n'}{1+\beta} \right) + \gamma \left(\frac{1}{1+\beta} - (nn') \right) s \right\}$$

$$= 2 (n' \varepsilon) \left\{ (n' s) \left(n - \frac{n'}{1+\beta} \right) + \left(\frac{1}{1+\beta} - (nn') \right) s \right\}$$

$$- \left(\frac{1}{\gamma'} + \frac{1}{\gamma} \right) \left((n [\varepsilon s]) \gamma [n' n] + \gamma' (n' [n \varepsilon]) [n' s] \right)$$

$$= - \frac{1}{\gamma} (2+\beta) \left((S[n\varepsilon]) \gamma[n'n] + \gamma'(\varepsilon[n'n])[n'S] \right)$$

$$= - (2+\beta) \left((S[n\varepsilon])[n'n] + \frac{1}{1+\beta} (\varepsilon[n'n])[n'S] \right)$$

$\therefore$ Total

$$= n \left[\beta \left(S, n - \frac{n'}{1+\beta} \right) (n'\varepsilon) + 2(n'\varepsilon)(n'S) \right] \quad \left| \quad \beta(Sn) - \frac{\beta}{1+\beta}(Sn') + 2(Sn)$$

$$+ \varepsilon \left[\frac{\beta K}{\gamma} (Sn') - \beta \left(S, n - \frac{n'}{1+\beta} \right) (nn') \right]$$

$$+ S \left[2(n'\varepsilon) \left(\frac{1}{1+\beta} - (nn') \right) - \frac{\beta K}{\gamma} (n'\varepsilon) \right]$$

$$- \frac{2(n'\varepsilon)(n'S)}{1+\beta} n' - [n'n] (2+\beta) (S, [n\varepsilon]) - [n'S] \frac{2+\beta}{1+\beta} (\varepsilon[n'n])$$

$$= n \left[(n'\varepsilon) \left\{ \beta(Sn) + \frac{2+\beta}{1+\beta} (n'S) \right\} \right]$$

$$+ \varepsilon \left[\cancel{(Sn)} \frac{\beta}{\alpha(1+\beta)} \cancel{(2+\beta+\alpha+\alpha\beta)} (Sn') - \beta(nn')(Sn) \right]$$

$$+ S(n'\varepsilon) \frac{\beta(2+\beta)}{1+\beta} - \frac{\alpha\beta}{\alpha(1+\beta)} (2+\beta+\alpha+\alpha\beta) (Sn') - \frac{2(n'\varepsilon)(n'S)}{1+\beta} n' - (2+\beta) (S, [n\varepsilon]) [n'n] - \frac{2+\beta}{1+\beta} [n'S] (\varepsilon[n'n])$$

$$= (n'S) \left[n(n'\varepsilon)\beta - \varepsilon(nn')\beta \right]$$

$$+ (n'S) \left[n(n'\varepsilon) \frac{2+\beta}{1+\beta} + \varepsilon \frac{\beta}{\alpha(1+\beta)} (2+\beta+\alpha+\alpha\beta) - \frac{2(n'\varepsilon)}{1+\beta} n' \right]$$

$$- ([Sn]\varepsilon) (2+\beta) [n'n] - [n'S] \frac{2+\beta}{1+\beta} (\varepsilon[n'n])$$

$$- S(n'\varepsilon) \frac{\beta(2+\beta)}{1+\beta}$$

1-Comp. $(n_1 S_1 + n_2 S_2 + n_3 S_3) \left[n_1 (n'\varepsilon)\beta + \varepsilon_1 (nn')\beta \right]$

$$+ (n'_1 S_1 + n'_2 S_2 + n'_3 S_3) \left[n_1 (n'\varepsilon) \frac{\beta+2}{1+\beta} + \varepsilon_1 \frac{\beta}{\alpha(1+\beta)} (2+\beta+\alpha+\alpha\beta) - \frac{2(n'\varepsilon)}{1+\beta} n'_1 \right]$$

$$+ \left\{ \left(\frac{S_2 n_3 - S_3 n_2}{n_2 S_3 - n_3 S_2} \right) \varepsilon_1 + (S_3 n_1 - S_1 n_3) \varepsilon_2 + (S_1 n_2 - S_2 n_1) \varepsilon_3 \right\} (2+\beta) [n'n],$$

$$\begin{aligned}
& - (n_2' S_3 - n_3' S_2) \frac{2+\beta}{1+\beta} (\mathcal{E}[n'n]) - S_1 (n'\mathcal{E}) \frac{\beta(2+\beta)}{1+\beta} \\
& = (n_1 S_1 + n_2 S_2) \left[n_1 (n'\mathcal{E})\beta + \mathcal{E}_1 (nn')\beta \right] \\
& + (n_1' S_1 + n_2' S_2) \left[n_1 (n'\mathcal{E}) \frac{\beta+2}{1+\beta} + \mathcal{E}_1 \frac{\beta}{\alpha(1+\beta)} (2+\beta+\alpha+\alpha\beta) - \frac{2(n'\mathcal{E})}{1+\beta} n_1' \right] \\
& + \left\{ S_2 n_3 \mathcal{E}_1 - S_1 n_3 \mathcal{E}_2 + (S_1 n_2 - S_2 n_1) \mathcal{E}_3 \right\} (2+\beta) [n'n], \\
& + n_3' \frac{2+\beta}{1+\beta} S_2 (\mathcal{E}[n'n]) - S_1 (n'\mathcal{E}) \frac{\beta(2+\beta)}{1+\beta} \\
& + S_3 \left\{ n_3 \left[n_1 (n'\mathcal{E})\beta + \mathcal{E}_1 (nn')\beta \right] + n_3' \left[n_1 (n'\mathcal{E}) \frac{\beta+2}{1+\beta} + \mathcal{E}_1 \frac{\beta}{\alpha(1+\beta)} (2+\beta+\alpha+\alpha\beta) - \frac{2(n'\mathcal{E})}{1+\beta} n_1' \right] \right. \\
& \quad \left. + n_1 \mathcal{E}_2 - \mathcal{E}_1 n_2 - n_2' \frac{2+\beta}{1+\beta} (\mathcal{E}[n'n]) \right\}
\end{aligned}$$

$$\begin{aligned}
\text{Total} & = (n_1 S_1 + n_2 S_2) \left[n_1 (n'\mathcal{E})\beta + \mathcal{E}_1 (nn')\beta \right] \\
& + (n_1' S_1 + n_2' S_2) \left[n_1 (n'\mathcal{E}) \frac{\beta+2}{1+\beta} + \mathcal{E}_1 \frac{\beta}{\alpha(1+\beta)} (2+\beta+\alpha+\alpha\beta) - \frac{2(n'\mathcal{E})}{1+\beta} n_1' \right] \\
& + (S_1 [n\mathcal{E}]_1 + S_2 [n\mathcal{E}]_2) (2+\beta) [n'n] - \cancel{S_1} - \frac{2+\beta}{1+\beta} (\mathcal{E}[n'n]) \cancel{S_1} \\
& - \frac{2+\beta}{1+\beta} (\mathcal{E}[n'n]) \cancel{S_2} - (n'\mathcal{E}) \frac{\beta(2+\beta)}{1+\beta} \times S_1 - (n'\mathcal{E}) \frac{\beta(2+\beta)}{1+\beta} \gamma S_2 \\
& + S_3 \left\{ n_3 \left[n_1 (n'\mathcal{E})\beta + \mathcal{E}_1 (nn')\beta \right] + n_3' \left[n_1 (n'\mathcal{E}) \frac{2+\beta}{1+\beta} + \mathcal{E}_1 \frac{\beta}{\alpha(1+\beta)} (2+\beta+\alpha+\alpha\beta) - \frac{2(n'\mathcal{E})}{1+\beta} n_1' \right] \right. \\
& \quad \left. - \frac{2+\beta}{1+\beta} (\mathcal{E}[n'n]) \right\} - \frac{2+\beta}{1+\beta} (\mathcal{E}[n'n]) l - (n'\mathcal{E}) \frac{\beta(2+\beta)}{1+\beta} z
\end{aligned}$$

* Suppose $u_3 v_3 = \frac{1}{(2\pi\hbar)^3}$, $u_4 = v_4 = 0$.

$\therefore S_3 = d$ $S_2 = -i S_1$

$$\begin{aligned}
& x(1, 0, 0) \\
& y(0, 1, 0) \\
& z(0, 0, 1) \\
& j(0, n_3', -n_2') \\
& k(-n_3', 0, n_1') \\
& l(n_2', -n_1', 0)
\end{aligned}$$

$$\therefore \text{Total} = S, C + i S, D + dB$$

$$\text{where } C = n_1 \left[n(n'\epsilon)\beta - \epsilon(nn')\beta \right] \\ + n_1' \left[n(n'\epsilon)\frac{\beta+2}{1+\beta} + \epsilon\frac{\beta}{\alpha(1+\beta)}(2+\beta+\alpha+\alpha\beta) - \frac{2(n'\epsilon)}{1+\beta}n' \right] \\ + [n\epsilon]_1(2+\beta)[n'n] - \frac{2+\beta}{1+\beta}(\epsilon[n'n]) - (n'\epsilon)\frac{\beta(2+\beta)}{1+\beta} \times$$

$$D = n_2 \left[n(n'\epsilon)\beta - \epsilon(nn')\beta \right] \\ + n_2' \left[n(n'\epsilon)\frac{\beta+2}{1+\beta} + \epsilon\frac{\beta}{\alpha(1+\beta)}(2+\beta+\alpha+\alpha\beta) - \frac{2(n'\epsilon)}{1+\beta}n' \right] \\ + [n\epsilon]_2(2+\beta)[n'n] - \frac{2+\beta}{1+\beta}(\epsilon[n'n]) - (n'\epsilon)\frac{\beta(2+\beta)}{1+\beta} \gamma$$

$$B = n_3 \left[n(n'\epsilon)\beta - \epsilon(nn')\beta \right] + \cancel{B_3} \left[n(n'\epsilon)\frac{\beta+2}{1+\beta} \right] \\ + n_3' \left[n(n'\epsilon)\frac{\beta+2}{1+\beta} + \epsilon\frac{\beta}{\alpha(1+\beta)}(2+\beta+\alpha+\alpha\beta) - \frac{2(n'\epsilon)}{1+\beta}n' \right] \\ + [n\epsilon]_3(2+\beta)[n'n] - \frac{2+\beta}{1+\beta}(\epsilon[n'n]) - (n'\epsilon)\frac{\beta(2+\beta)}{1+\beta} z$$

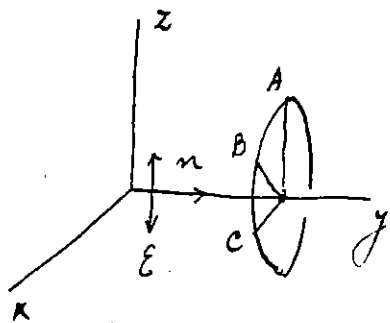
$$H' = \frac{(2\pi h)^3 e^2 v'}{2mc^2(\nu - \nu' + \frac{2mc^2}{h})} \left\{ d(A - iB) + S, (D + iC) \right\} e^{i\nu'(t - \frac{z}{c})}$$

complex conjugate term

$$A = - \frac{\left\{ 2 + \alpha(1 - (nn')) \right\}^2}{\alpha \left\{ 1 + \alpha(1 - (nn')) \right\}} \left[n', \epsilon - n \cdot \frac{\alpha^2(1 - (nn'))}{\left\{ 2 + \alpha(1 - (nn')) \right\}^2} (n'\epsilon) \right]$$

For linearly polarised light A, B, C & D are real.

$d(A - iB)$ & $S, (D + iC)$, each constitutes ^{an} elliptically polarised ~~and~~ light, which is incoherent ^{with} each other.



$$n_1=0, n_2=1, n_3=0$$

$$E_1=0, E_2=0, E_3=E$$

$$\left. \begin{array}{l} \textcircled{A} \quad n_1'=0, n_2'=0, n_3'=1 \\ \textcircled{B} \quad n_1'=\frac{1}{\sqrt{2}}, n_2'=0, n_3'=\frac{1}{\sqrt{2}} \\ \textcircled{C} \quad n_1'=1, n_2'=0, n_3'=0 \end{array} \right\} (nn')=0$$

$$\textcircled{A} \quad A = - \frac{\{2+\alpha\}^2}{\alpha(1+\alpha)} \left[n', \epsilon - n \cdot \frac{\alpha^2}{(2+\alpha)^2} (n' \epsilon) \right] \quad [n', n]$$

$$= + \frac{\alpha}{1+\alpha} \epsilon \quad \text{in direction of } x$$

$$B = \epsilon^2 \frac{2+\alpha}{1+\alpha} + \epsilon^2 \frac{\alpha}{1+\alpha} (2+2\alpha+\alpha^2) - \frac{2\epsilon^2}{1+\alpha} - \epsilon \frac{\alpha(2+\alpha)}{1+\alpha} \epsilon$$

$$= \left| \epsilon \frac{2+\alpha}{1+\alpha} \right|_y + \left| \epsilon \frac{\alpha(2+\alpha)}{1+\alpha} \right|_z - \left| \epsilon \frac{\alpha(2+\alpha)}{1+\alpha} \right|_z = \left| \epsilon \frac{2+\alpha}{1+\alpha} \right|_y$$

$$C = \left| \epsilon(2+\alpha) \right|_x - \left| \epsilon \frac{\alpha(2+\alpha)}{1+\alpha} \right|_x = \left| \epsilon \frac{(2+\alpha)}{1+\alpha} \right|_x$$

$$D = \left| \alpha \epsilon \right|_y - \left| \epsilon \frac{\alpha(2+\alpha)}{1+\alpha} \right|_y = - \left| \epsilon \frac{\alpha}{1+\alpha} \right|_y$$

$$A - iB = \left| \frac{\alpha}{1+\alpha} \epsilon \right|_x - i \left| \epsilon \frac{2+\alpha}{1+\alpha} \right|_y$$

$$-(D + iC) = \left| \frac{\alpha}{1+\alpha} \epsilon \right|_y - i \left| \epsilon \frac{2+\alpha}{1+\alpha} \right|_x$$

$$\textcircled{C} \quad A = \left| + \frac{(2+\alpha)^2}{\alpha(1+\alpha)} \frac{\alpha^2 \epsilon}{(2+\alpha)^2} \epsilon \right|_z = \frac{\epsilon \alpha}{1+\alpha} - \left| \frac{(2+\alpha)^2}{\alpha(1+\alpha)} \epsilon \right|_y + \left| \epsilon \frac{\alpha}{1+\alpha} \right|_z$$

$$- \left| \frac{(2+\alpha)^2}{\alpha(1+\alpha)} \epsilon \right|_y +$$

$$\frac{3.9}{1.7} = 2.35^2$$

$$\frac{1+(1.7)^2}{1.7} = \frac{3}{2.3}$$

$$d=1.5 \quad \frac{1+(1.5)^2}{(1.5)^2} = \frac{3.25}{1.5} = 2.16$$

$$\frac{1+(1.3)^2}{1.3} = \frac{2.69}{1.3} = 2.07$$

$$\frac{1+(1+\alpha)^2}{(1+\alpha)^2} \epsilon^2 - 2\epsilon^2$$

$$\frac{0.7}{2.07} = \frac{1.67}{3.3} = 1.35$$

$$.65 - 2 = .65$$

$$\frac{1+(1.4)^2}{1.4} = 2.11 - 2 = .11$$

$$1 \quad 4.5 \quad \frac{1+(1.45)^2}{1.45} = 2.11 - 2 = .11$$

$$1.96 \quad \frac{2.96}{1.4} = 2.11 - 2 = .11$$

6.

$$B = \left| \varepsilon(2+\alpha) \right|_y + \left| \frac{2+\alpha}{1+\alpha} \varepsilon \right|_y = \varepsilon \left| \frac{(2+\alpha)^2}{1+\alpha} \right|_y$$

$$C = \left| \varepsilon \frac{1}{1+\alpha} (2+2\alpha+\alpha^2) \right|_y$$

$$D = - \left| \frac{2+\alpha}{1+\alpha} \varepsilon \right|_z$$

$$A - iB = - \left| \frac{(2+\alpha)^2}{\alpha(1+\alpha)} \varepsilon \right|_y + \left| \varepsilon \frac{\alpha}{1+\alpha} \right|_z - i \left| \varepsilon \frac{(2+\alpha)^2}{1+\alpha} \right|_y$$

$$-(D + iC) = \left| \frac{2+\alpha}{1+\alpha} \varepsilon \right|_z - i \left| \varepsilon \frac{2+2\alpha+\alpha^2}{1+\alpha} \right|_y$$

[191-ii]

$$\mathcal{E}^2 \text{-term} = \frac{4K}{\alpha v} \left\{ \frac{v}{v'} + \frac{v'}{v} \right\} \mathcal{E}^2$$

$$(n'\mathcal{E})^2 \text{-term} = -\frac{8K}{\alpha v} (n'\mathcal{E})^2$$

$$I = \frac{e^4}{2m^2 c^4 \hbar^2} \left(\frac{v'}{v} \right)^3 \left\{ 2 \left(\frac{v}{v'} + \frac{v'}{v} \right) \mathcal{E}^2 \right.$$

$$I = \frac{e^4}{m^2 c^4 \hbar^2} \left(\frac{v'}{v} \right)^3 \cdot \frac{1}{8} \frac{\alpha v}{K} \left[\frac{4K}{\alpha v} \left(\frac{v}{v'} + \frac{v'}{v} \right) \mathcal{E}^2 - \frac{8K}{\alpha v} (n'\mathcal{E})^2 \right]$$

$$= \frac{e^4}{2m^2 c^4 \hbar^2} \left(\frac{v'}{v} \right)^3 \left[\left(\frac{v}{v'} + \frac{v'}{v} \right) \mathcal{E}^2 - \cancel{\frac{2K}{\alpha v}} 2 (n'\mathcal{E})^2 \right]$$

$$\frac{v'}{v} = \frac{1}{1 + \alpha(1 - (nn'))} \quad \frac{v}{v'} = 1 + \alpha(1 - (nn'))$$

$$\therefore \frac{v'}{v} + \frac{v}{v'} = \frac{1 + (1 + \alpha(1 - (nn')))^2}{1 + \alpha(1 - (nn'))}$$

$$\therefore I = \frac{e^4}{2m^2 c^4 \hbar^2} \left(\frac{1}{1 + \alpha(1 - (nn'))} \right)^3 \left\{ \frac{1 + (1 + \alpha(1 - (nn')))^2}{1 + \alpha(1 - (nn'))} \mathcal{E}^2 - 2(n'\mathcal{E})^2 \right\} \cancel{\frac{1}{2m^2 c^4 \hbar^2}}$$

[191 - ii]

$$\mathcal{E}^2 \text{-term} = \frac{4K}{\alpha v} \left\{ \frac{v}{v'} + \frac{v'}{v} \right\} \mathcal{E}^2$$

$$(n'\mathcal{E})^2 \text{-term} = -\frac{8K}{\alpha v} (n'\mathcal{E})^2$$

$$I = \frac{e^4}{2m^2 c^4 \lambda^2} \left(\frac{v'}{v} \right)^3 \left\{ 2 \left(\frac{v}{v'} + \frac{v'}{v} \right) \mathcal{E}^2 \right.$$

$$I = \frac{e^4}{m^2 c^4 \lambda^2} \left(\frac{v'}{v} \right)^3 \cdot \frac{1}{8} \frac{\alpha v}{K} \left[\frac{4K}{\alpha v} \left(\frac{v}{v'} + \frac{v'}{v} \right) \mathcal{E}^2 - \frac{8K}{\alpha v} (n'\mathcal{E})^2 \right]$$

$$= \frac{e^4}{2m^2 c^4 \lambda^2} \left(\frac{v'}{v} \right)^3 \left[\left(\frac{v}{v'} + \frac{v'}{v} \right) \mathcal{E}^2 - \cancel{\frac{2K}{\alpha v}} 2 (n'\mathcal{E})^2 \right]$$

$$\frac{v'}{v} = \frac{1}{1 + \alpha(1 - (nn'))} \quad \frac{v}{v'} = 1 + \alpha(1 - (nn'))$$

$$\therefore \frac{v'}{v} + \frac{v}{v'} = \frac{1 + (1 + \alpha(1 - (nn')))^2}{1 + \alpha(1 - (nn'))}$$

$$\therefore I = \frac{e^4}{2m^2 c^4 \lambda^2} \left(\frac{1}{1 + \alpha(1 - (nn'))} \right)^3 \left\{ \frac{1 + (1 + \alpha(1 - (nn')))^2}{1 + \alpha(1 - (nn'))} \mathcal{E}^2 - 2(n'\mathcal{E})^2 \right\} \cancel{\frac{1}{2m^2 c^4 \lambda^2}}$$

~~Q₂α~~

$$n = E \times H$$

$$J = e e \varphi' \alpha \psi$$

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$$J = e s = e e \varphi' \alpha \psi$$

$$J_m = e e \int d m' \left\{ G_2 \alpha u' + v \alpha F_1' \right\} e^{\frac{i}{\hbar} [(E + \hbar \nu - E')t - (M + \frac{\hbar \nu}{c} n - m')r]} \\ + \left\{ G_1' \alpha u + v' \alpha F_2 \right\} e^{-\frac{i}{\hbar} [(E + \hbar \nu - E')t - (M + \frac{\hbar \nu}{c} n - m')r]} \right\}$$

$$Q_m = \frac{e}{R} \int d v \int d m'$$

$$= \frac{e}{R} (2\pi \hbar)^3 \left\{ (G_2 \alpha u' + v \alpha F_1') e^{i v' (t - \frac{R}{c})} + (G_1' \alpha u + v' \alpha F_2) e^{-i v' (t - \frac{R}{c})} \right\}$$

$$G_2 = \frac{e}{2 m c \hbar \nu} v [(Q H) - (P E)] \quad Q = \hbar \alpha \quad P = i \hbar \alpha$$

$$F_1' = \frac{-e}{2 m c \hbar \nu'} [2(\alpha M') + (Q H) + (P E)] u$$

$$G_2 \alpha u' = \frac{e}{2 m c \hbar \nu} v [\hbar (\alpha H) - i \hbar (\alpha \tilde{P})] \alpha u' \\ = \frac{e \hbar}{2 m c \hbar \nu} v [(\alpha H) \alpha - i (\tilde{P} \alpha) \alpha] u'$$

$$v \alpha F_1' = \frac{-e v \alpha}{2 m c \hbar \nu'} [2(\alpha M') + \hbar (\tilde{Q} H) + i \hbar (Q E)] u' \\ = \frac{-e \hbar v}{2 m c \hbar \nu'} \left[\frac{2(\alpha M')}{\hbar} \alpha + \alpha (\alpha H) + i \alpha (\alpha E) \right] u'$$

$$\left[\frac{E'}{c} + \beta_1 (\alpha M') + \beta_3 m c \right] u' = 0$$

$$v \left[\frac{E'}{c} (\alpha H) \alpha + (\alpha H) \alpha (\alpha M') + (\alpha H) \alpha \beta_3 m c \right] u' = 0. \quad \begin{matrix} \beta_3 \tilde{\alpha} (\alpha H) \\ - \alpha (\alpha H) \beta_3 \end{matrix}$$

$$v [1 + \beta_3] = 0 \quad \therefore v [(\alpha H) \alpha - (\alpha H) \alpha \beta_3] = 0 \quad \therefore v (\alpha H) \alpha \beta_3 = + v (\alpha H) \alpha$$

$$\therefore v \left[\left(\frac{E'}{c} + m c \right) (\alpha H) \alpha \right] u' = - v [(\alpha H) \alpha (\alpha M')] u'$$

$$(\alpha H) \alpha = (\alpha_1 H_1 + \alpha_2 H_2 + \alpha_3 H_3) \alpha = H_1 - i \alpha_3 H_2 + i \alpha_2 H_3 = H_1 + i [\alpha H]^2$$

$$\underline{(\alpha H) \alpha = H + i [\alpha H]}$$

$$\underline{\alpha(\alpha H) = H - i [\alpha H]}$$

$$\cancel{(\alpha H) \alpha} \quad \therefore (\alpha H) \alpha + \alpha(\alpha H) = 2H$$

$$\therefore [(\alpha H) \alpha(\alpha M')] = [\{ 2H - \alpha(\alpha H) \} (\alpha M')] = 2H(\alpha M') - \alpha(\alpha H)(\alpha M')$$

$$= 2H(\alpha M') - \alpha(H M') - i \alpha(\alpha, H \times M')$$

$$= 2H(\alpha M') - \alpha(H M') - i [H \times M'] - [\alpha, [H \times M']]$$

$$= 2H(\alpha M') - \alpha(H M') - i [H \times M'] - H(\alpha M') + M'(\alpha H)$$

$$= H(\alpha M') - \alpha(H M') - i [H \times M'] + M'(\alpha H)$$

$$\textcircled{x} \therefore \overset{v'}{V} [(\alpha H) \alpha] \overset{v'}{u}' = \frac{+1}{\frac{E'}{c} + mc} \overset{v'}{V} \left[-H(\alpha M') + \alpha(H M') + i [H \times M'] + M'(\alpha H) \right] \overset{v'}{u}'$$

$$- \overset{v'}{V} \left[\left(\frac{E'}{c} + mc \right) \alpha(\alpha H) \right] \overset{v'}{u}' = + \overset{v'}{V} [\alpha(\alpha H)(\alpha M')] \overset{v'}{u}'$$

$$\textcircled{x} \therefore - \overset{v'}{V} [\alpha(\alpha H)] \overset{v'}{u}' = \frac{1}{\frac{E'}{c} + mc} \overset{v'}{V} \left[H(\alpha M') + \alpha(H M') + i [H \times M'] + \overset{v'}{M'(\alpha H)} \right] \overset{v'}{u}'$$

$$\overset{v'}{V} \left[\frac{E'}{c} \alpha + \alpha(\alpha M') + \alpha \beta_1 \beta_3 mc \right] \overset{v'}{u}' = 0$$

$$\overset{v'}{V} [\alpha \beta_1 + \beta_3 \alpha \beta_1] = 0$$

$$\overset{v'}{V} (\alpha \beta_1) - \overset{v'}{V} \beta_1 \alpha \beta_3 = 0$$

$$\overset{v'}{V} (\alpha) = \overset{v'}{V} \alpha \beta_3$$

$$\overset{v'}{V} \left[\left(\frac{E'}{c} + mc \right) \alpha \right] = - \overset{v'}{V} [\alpha(\alpha M')] \overset{v'}{u}'$$

$$= - \overset{v'}{V} [M' - i [\alpha M']]$$

$$\textcircled{x} \therefore - \overset{v'}{V} \alpha = \frac{1}{\frac{E'}{c} + mc} [-i [\alpha M'] + M']$$

$$-i(\alpha E)\alpha = (\alpha E)\alpha = -iE \frac{\gamma}{c} [\alpha E]$$

$$-\frac{i\gamma}{c}a = E.$$

$$-i\alpha(\alpha E) = \alpha(\alpha E) = -iE \frac{\gamma}{c} [\alpha E]$$

$$a\gamma = +ice.$$

$$G_2 \alpha u' + v \alpha F_1' = \frac{e}{2mc\gamma\gamma'} V \left[\gamma'(\alpha H)\alpha - \gamma\alpha(\alpha H) - i\gamma'(\alpha E)\alpha - i\gamma\alpha(\alpha E) - 2\frac{\gamma(\alpha M')}{h} \alpha \right] u'$$

$$= \frac{e}{2mc\gamma\gamma'} V \left[\frac{1}{\frac{E'}{c} + mc} \left\{ H(\alpha M')(\gamma - \gamma') + \alpha(HM')(\gamma + \gamma') + i[H \times M'](\gamma + \gamma') - M'(\alpha H)(\gamma + \gamma') + \frac{2(\alpha M')\gamma}{h} (M' - i[\alpha M']) \right\} - iE(\gamma + \gamma') - [\alpha E](\gamma - \gamma') \right] u'$$

$$\frac{1}{\frac{E'}{c} + mc} \left[i[H \times M'](\gamma + \gamma') + \frac{2ic(\alpha M')M'}{h} - i(\frac{E'}{c} + mc)E(\gamma + \gamma') \right]$$

$$[H \times M'] = \left[H \times \left(\frac{h\gamma}{c} n - \frac{h\gamma'}{c} n' \right) \right] = \frac{h\gamma}{c} [H \times n] - \frac{h\gamma'}{c} [H \times n'] = \frac{h\gamma}{c} E - \frac{h\gamma'}{c} [H \times n']$$

$$= \frac{h\gamma}{c} E + \frac{h\gamma'}{c} \{ n(n'E) - E(nn') \}.$$

$$(\frac{E'}{c} + mc)E = (2mc + \frac{h\gamma}{c} - \frac{h\gamma'}{c})E = 2mcE + \frac{h\gamma}{c}E - \frac{h\gamma'}{c}E$$

$$2(\alpha M')M' = (E, \frac{h\gamma}{c}n - \frac{h\gamma'}{c}n')M' = -\frac{h\gamma'}{c}(n'E) \left(\frac{h\gamma}{c}n - \frac{h\gamma'}{c}n' \right)$$

$$= -\frac{h^2}{c^2}\gamma\gamma' n(n'E) + \frac{h^2}{c^2}\gamma\gamma' n^2(n'E)$$

$$[H \times M'](\gamma + \gamma') = \frac{h\gamma}{c}E(\gamma + \gamma') + \frac{h\gamma'}{c}(\gamma + \gamma') \left\{ n(n'E) - E(nn') \right\}$$

$$\frac{h\gamma\gamma}{c} n(n'E) + \frac{h\gamma'}{c} n(n'E)$$

$$-(\frac{E'}{c} + mc) \mathcal{E}(\nu + \nu') = -2mc(\nu + \nu') \mathcal{E} - \frac{h\nu}{c}(\nu + \nu') \mathcal{E} + \frac{h\nu'}{c}(\nu + \nu') \mathcal{E} \quad \text{B.4}$$

$$\frac{2c(\mathcal{E}M')M'}{h} = -2 \frac{h\nu'}{c} n(n' \mathcal{E}) + 2 \frac{h\nu'^2}{c} n'(n' \mathcal{E})$$

$$\therefore \frac{i}{\frac{E'}{c} + mc} \left[[H \times M'](\nu + \nu') + \frac{2c(\mathcal{E}M')M'}{h} - (\frac{E'}{c} + mc) \mathcal{E}(\nu + \nu') \right]$$

$$= \frac{i}{\frac{E'}{c} + mc} \left[\frac{h\nu'}{c} n(n' \mathcal{E}) (\nu - \nu') + (\nu + \nu') \left\{ \frac{h\nu'}{c} - \frac{h\nu'}{c}(nn') - 2mc \right\} \mathcal{E} \right] \quad \text{B.5}$$

$$= \frac{i}{\frac{E'}{c} + mc} \left[\frac{h\nu'}{c} n(n' \mathcal{E}) (\nu - \nu') - \frac{(\nu + \nu')^2}{\nu} mc \mathcal{E} + \frac{2h\nu'^2}{c} n'(n' \mathcal{E}) \right]$$

$$\quad \quad \quad + \frac{2h\nu'}{c} n'(n' \mathcal{E})$$

$$\nu' = \frac{\nu}{1 + \frac{h\nu}{mc\nu}(1 - (nn'))}$$

$$\nu' + \frac{h\nu\nu'}{mc^2}(1 - (nn')) = \nu$$

$$\frac{\nu'}{\nu} mc + \frac{h\nu'}{mc} (1 - (nn')) = mc$$

$$\therefore \frac{h\nu'}{mc} (1 - (nn')) - 2mc = (\frac{\nu'}{\nu} + 1)mc = (\frac{\nu + \nu'}{\nu})mc$$

$$\therefore \mathcal{O}_m = \frac{e}{R} (2\pi\hbar)^3 \frac{e}{2mc\nu\nu'} \frac{1}{\frac{E'}{c} + mc} \left[H(\mathcal{A}M')(\nu - \nu') + \mathcal{A}(HM')(\nu + \nu') - \right.$$

$$\quad \quad \quad - M'(\mathcal{A}H)(\nu + \nu') - i \frac{2(\mathcal{A}M')\nu}{\hbar} [\mathcal{A} \times M']$$

$$\quad \quad \quad - (\frac{E'}{c} + mc)(\nu - \nu') [\mathcal{A} \mathcal{E}] + u'$$

$$\quad \quad \quad + i \left\{ \frac{h\nu'}{c} n(n' \mathcal{E}) (\nu - \nu') - \frac{(\nu + \nu')^2}{\nu} mc \mathcal{E} + \frac{2h\nu'^2}{c} n'(n' \mathcal{E}) \right\}$$

$$\overline{\mathcal{O}_m} = \frac{e^2}{R} (2\pi\hbar)^3 \frac{i}{2mc\nu\nu' (2mc + \frac{h}{c}(\nu - \nu'))} \left\{ \frac{h\nu'}{c} n(n' \mathcal{E}) (\nu - \nu') - \frac{(\nu + \nu')^2}{\nu} mc \mathcal{E} + \frac{2h\nu'^2}{c} n'(n' \mathcal{E}) \right\} u'$$

$$\overline{H}_M = \frac{e^2}{2mc^2 R} \frac{1}{\gamma(2mc + \frac{h}{c}(\gamma - \gamma'))} \left\{ \frac{h\gamma'}{c} [n' \times n] (n' \cdot \underline{E}) (\gamma - \gamma') \pm mc [n' \times \underline{E}] \frac{(\gamma + \gamma')^2}{\gamma} \right\} \underline{u}$$

$$H(\alpha M')(\gamma - \gamma') = (\gamma - \gamma') \left\{ \frac{h\gamma}{c} H(n\alpha) - \frac{h\gamma'}{c} H(n'\alpha) \right\}$$

$$\alpha(HM')(\gamma + \gamma') = -(\gamma + \gamma') \frac{h\gamma'}{c} \alpha(n'H)$$

$$-M'(\alpha H)(\gamma + \gamma') = -(\gamma + \gamma') \left\{ \frac{h\gamma}{c} n(\alpha H) - \frac{h\gamma'}{c} n'(\alpha H) \right\}$$

$$-[\alpha \underline{E}] \left(\frac{E'}{c} + mc \right) (\gamma - \gamma') = -\frac{1}{h} [\alpha \underline{E}] M'^2 = -\left\{ H(n\alpha) - n(\alpha H) \right\} \left(\left(\frac{h\gamma}{c} \right)^2 + \left(\frac{h\gamma'}{c} \right)^2 - 2 \frac{h^2 \gamma \gamma'}{c^2} (nn') \right) M'^2$$

$$-i \frac{2(\alpha M')\gamma}{h} [\alpha \times M'] = +i \frac{2h\gamma\gamma'}{h^2 c} (\alpha n') [\alpha M'] = -2\gamma'(n'E) [\alpha M']$$

$$G_1' \alpha u = \frac{-e}{2mch\gamma'} \gamma' \left\{ 2(\tilde{\alpha} M') - (P\tilde{E}) + (Q\tilde{H}) \right\} \alpha u$$

$$= \frac{-e}{2mch\gamma'} \gamma' \left\{ 2(\tilde{\alpha} M') - i\hbar(\alpha \tilde{E}) + \hbar(\alpha \tilde{H}) \right\} \alpha u$$

$$= \frac{-e}{2mch\gamma'} \gamma' \left\{ 2(\tilde{\alpha} M') \alpha - i\hbar(\alpha \tilde{E}) \alpha + (\alpha \tilde{H}) \alpha \right\} u$$

$$\gamma' \alpha F_2 = \frac{e}{2mch\gamma} \gamma' \alpha \left\{ (P\tilde{E}) + (Q\tilde{H}) \right\} u$$

$$= \frac{e}{2mch\gamma} \gamma' \alpha \left\{ i(\alpha \tilde{E}) + (\alpha \tilde{H}) \right\} u$$

$$= \frac{e}{2mch\gamma} \gamma' \left\{ i\alpha(\alpha \tilde{E}) + \alpha(\alpha \tilde{H}) \right\} u$$

$$\gamma' \left[\frac{E'}{c} + \beta_1 (\alpha M') + \beta_3 mc \right] = 0 \quad [1 + \beta_3] u = 0 \quad [(\alpha \tilde{H}) \alpha - \beta_3 (\alpha \tilde{H}) \alpha] u = 0$$

$$\gamma' \left[\frac{E'}{c} (\alpha \tilde{H}) \alpha + (\alpha M') (\alpha \tilde{H}) \alpha + mc \beta_3 (\alpha \tilde{H}) \alpha \right] u = 0$$

$$\gamma' \left[\left(\frac{E'}{c} + mc \right) (\alpha \tilde{H}) \alpha + (\alpha M') (\alpha \tilde{H}) \alpha \right] u = 0$$

$$\therefore -\gamma' \gamma \left[(\alpha \tilde{H}) \alpha \right] u = \frac{1}{\frac{E'}{c} + mc} \gamma' \left[(\alpha M') (\alpha \tilde{H}) \alpha \right] u$$

$$(\alpha M')(\alpha \tilde{H}) = (M' \tilde{H}) + i(\alpha, M' \times H)$$

$$\begin{aligned} \therefore (\alpha M')(\alpha \tilde{H}) \alpha &= \alpha(M' \tilde{H}) + i[M' \times \tilde{H}] - [\alpha[M' \times \tilde{H}]] \\ &= \alpha(M' \tilde{H}) + i[M' \times \tilde{H}] - M'(\alpha \tilde{H}) + \tilde{H}(\alpha M'). \end{aligned}$$

$$\textcircled{x} \therefore -V' V [\alpha \tilde{H}] \alpha u = \frac{V}{\frac{E'}{c} + mc} V' \left[\alpha(M' \tilde{H}) - i[H \times M'] - M'(\alpha \tilde{H}) + \tilde{H}(\alpha M') \right] u$$

$$\textcircled{x} V' \left[\frac{E'}{c} \alpha(\alpha \tilde{H}) + (\alpha M') \alpha(\alpha \tilde{H}) + mc \beta_3 \alpha(\alpha \tilde{H}) \right] u = 0.$$

$$\therefore V' V' [\alpha(\alpha \tilde{H})] u = \frac{V'}{\frac{E'}{c} + mc} V' \left[-(\alpha M') \alpha(\alpha \tilde{H}) \right] u.$$

$$\begin{aligned} -(\alpha M') \alpha(\alpha \tilde{H}) &= -(\alpha M') (2H - (\alpha H) \alpha) = -2H(\alpha M') + (\alpha M')(\alpha H) \alpha \\ &= -2\tilde{H}(\alpha M') + \alpha(M' \tilde{H}) - i[\tilde{H} \times M'] - M'(\alpha \tilde{H}) + \tilde{H}(\alpha M'). \end{aligned}$$

$$\textcircled{x} \therefore V' V' [\alpha(\alpha \tilde{H})] u = \frac{V'}{\frac{E'}{c} + mc} V' \left[-2\tilde{H}(\alpha M') + \alpha(M' \tilde{H}) - i[\tilde{H} \times M'] - M'(\alpha \tilde{H}) \right] u$$

$$V' \left[\frac{E'}{c} \alpha + (\alpha M') \alpha + \beta_3 \beta_1 \alpha mc \right] u = 0.$$

$$\begin{aligned} \textcircled{x} - V' V [\alpha] u &= \frac{V}{\frac{E'}{c} + mc} V' \left[(\alpha M') \alpha \right] u \\ &= \frac{V}{\frac{E'}{c} + mc} V' \left[M' + i[\alpha M'] \right] u. \end{aligned}$$

$$\textcircled{x} i V V' (\alpha \tilde{E}) \alpha u = i V V' (\alpha \tilde{E}) \alpha = V V' \{ i \tilde{E} - [\alpha \tilde{E}] \} u$$

$$\textcircled{x} i V' V' \alpha(\alpha \tilde{E}) = i V' V' \alpha(\alpha \tilde{E}) = V' V' \{ i \tilde{E} + [\alpha \tilde{E}] \} u.$$

$$\begin{aligned} G_2 \alpha u + V \alpha F_1' &= \frac{e}{2mc} \frac{V'}{V} \left\{ V' \alpha(\alpha \tilde{H}) - V(\alpha \tilde{H}) \alpha + i V' \alpha(\alpha \tilde{E}) + i V(\alpha \tilde{E}) \alpha \right. \\ &\quad \left. - 2V \frac{(\alpha M')}{h} \alpha \right\} u. \end{aligned}$$

$$= \frac{e}{2mcv\nu'} \nu' \left[\frac{1}{\frac{E'}{c} + mc} \left\{ \tilde{H}(\alpha M')(\nu - \nu') + \alpha(M' \tilde{H})(\nu + \nu') - i[\tilde{H} \times M'](\nu + \nu') \right. \right. \\ \left. \left. - M'(\alpha \tilde{H})(\nu + \nu') + 2 \frac{\nu(\tilde{\alpha} M')}{h} \{ M' + i[\alpha M'] \} \right. \right. \\ \left. \left. + i \tilde{E}(\nu + \nu') - [\alpha \tilde{E}](\nu - \nu') \right\} \right] u.$$

$$\bar{J} = \frac{c}{4\pi} [\epsilon_0 H] = \frac{c}{4\pi} |H|^2 = \frac{c}{4\pi} \frac{\nu'^2}{c^2} ([n' \alpha] [n' \tilde{\alpha}]) \\ = \frac{\nu'^2}{4\pi c} \{ (\alpha \tilde{\alpha}) - (n' \alpha)(n' \tilde{\alpha}) \}$$

$$\alpha_m = \alpha e^{i\nu(t - \frac{R}{c})} + \tilde{\alpha} e^{-i\nu(t - \frac{R}{c})}$$

$$\tilde{\alpha}_m = \tilde{\alpha} e^{-i\nu(t - \frac{R}{c})} + \alpha e^{+i\nu(t - \frac{R}{c})}.$$

$$\therefore \overline{\alpha_m \tilde{\alpha}_m} = 2 \alpha \tilde{\alpha}. \quad (n' \alpha_m)(n' \tilde{\alpha}_m) = 2(n' \alpha)(n' \tilde{\alpha}).$$

$$S = \chi + \xi k_1 + \eta k_2 + \zeta k_3 \quad ik_n = \sigma_n \quad \text{or} \quad k_n = -i \sigma_n$$

$$\therefore S = \chi - i(\xi \sigma_1 + \eta \sigma_2 + \zeta \sigma_3)$$

$$\text{Put } S = \alpha_0 - i \sum_k^3 b_k \sigma_k \quad S^{-1} = \alpha_0 + i \sum_k^3 b_k \sigma_k \quad \therefore SS^{-1} = \alpha_0^2 + \sum_k^3 b_k^2 = 1.$$

$$\left. \begin{aligned} b_0 &= \cos \frac{1}{2} \theta \cos \frac{1}{2} (\psi + \phi) \\ b_1 &= \sin \frac{1}{2} \theta \sin \frac{1}{2} (\psi - \phi) \\ b_2 &= \sin \frac{1}{2} \theta \cos \frac{1}{2} (\psi - \phi) \\ b_3 &= \cos \frac{1}{2} \theta \sin \frac{1}{2} (\psi + \phi) \end{aligned} \right\}$$

$$\begin{aligned} S \sigma_1 S^{-1} &= (\alpha_0 - i b_1 \sigma_1 - i b_2 \sigma_2 - i b_3 \sigma_3) \sigma_1 (\alpha_0 + i b_1 \sigma_1 + i b_2 \sigma_2 + i b_3 \sigma_3) \\ &= \sigma_1 (\alpha_0 - i b_1 \sigma_1 + i b_2 \sigma_2 + i b_3 \sigma_3) (\alpha_0 + i b_1 \sigma_1 + i b_2 \sigma_2 + i b_3 \sigma_3) \\ &= \sigma_1 \{ \alpha_0^2 + b_1^2 - b_2^2 - b_3^2 + 2i \alpha_0 b_2 \sigma_2 + 2i \alpha_0 b_3 \sigma_3 + 2i b_1 b_2 \sigma_3 - 2i b_1 b_3 \sigma_2 \} \\ &= (\alpha_0^2 + b_1^2 - b_2^2 - b_3^2) \sigma_1 - 2(\alpha_0 b_2 - b_1 b_3) \sigma_3 + 2(\alpha_0 b_3 + b_1 b_2) \sigma_2 \end{aligned}$$

$$\begin{aligned} \alpha_0^2 + b_1^2 - b_2^2 - b_3^2 &= \cos^2 \frac{1}{2} \theta \cos^2 \frac{1}{2} (\psi + \phi) + \sin^2 \frac{1}{2} \theta \sin^2 \frac{1}{2} (\psi - \phi) \\ &\quad - \sin^2 \frac{1}{2} \theta \cos^2 \frac{1}{2} (\psi - \phi) - \cos^2 \frac{1}{2} \theta \sin^2 \frac{1}{2} (\psi + \phi) \end{aligned}$$

$$\begin{aligned} &= \cos^2 \frac{1}{2} \theta \left\{ \cos^2 \frac{1}{2} (\psi + \phi) - \sin^2 \frac{1}{2} (\psi + \phi) \right\} \\ &\quad + \sin^2 \frac{1}{2} \theta \left\{ \sin^2 \frac{1}{2} (\psi - \phi) - \cos^2 \frac{1}{2} (\psi - \phi) \right\} \end{aligned}$$

$$= \cos^2 \frac{1}{2} \theta \cdot \cos (\psi + \phi) - \sin^2 \frac{1}{2} \theta \cos (\psi - \phi)$$

$$= \cos \psi \cos \phi - \cos \theta \cos \psi \cos \phi - \sin \psi \sin \phi$$

$$(\alpha_0^2 + b_1^2 - b_2^2 - b_3^2)^2 = \cos^2 \theta \cos^2 \psi \cos^2 \phi + \sin^2 \psi \sin^2 \phi - \frac{1}{2} \cos \theta \sin 2\psi \sin 2\phi$$

$$\left(\frac{\alpha_0^2 + b_1^2 - b_2^2 - b_3^2}{2} \right)^2 = \frac{1}{8} + \frac{1}{4} = \frac{3}{8} \sigma_1$$

$$2(b_0 b_2 - b_1 b_3) = 2 \left(\sin \frac{1}{2} \theta \cos \frac{1}{2} \theta \sin \frac{1}{2} (\psi - \varphi) \cos \frac{1}{2} (\psi + \varphi) \right. \\ \left. - \sin \frac{1}{2} \theta \cos \frac{1}{2} \theta \cos \frac{1}{2} (\psi - \varphi) \sin \frac{1}{2} (\psi + \varphi) \right) \\ = 2 - \sin \theta \sin \varphi$$

$$\therefore 2(b_0 b_2 - b_1 b_3)^2 = \frac{1}{4}.$$

$$2(b_0 b_3 + b_1 b_2) = \frac{1}{4} \cos^2 \frac{1}{2} \theta \sin (\psi + \varphi) + \frac{1}{4} \sin^2 \frac{1}{2} \theta \sin (\psi - \varphi) \\ = \frac{1}{4} \sin \psi \cos \varphi + \frac{1}{4} \cos \theta \cos \psi \sin \varphi.$$

$$\frac{2(b_0 b_3 + b_1 b_2)^2}{2(b_0 b_3 + b_1 b_2)^2} = \sin^2 \psi \cos^2 \varphi + \cos^2 \theta \cos^2 \psi \sin^2 \varphi + \frac{1}{2} \cos \theta \sin 2\psi \sin 2\varphi$$

$$\frac{2(b_0 b_3 + b_1 b_2)^2}{2(b_0 b_3 + b_1 b_2)^2} = \frac{3}{8}.$$

$$\therefore \text{Total} = 1.$$

$$\therefore (v \alpha u') (v' \alpha u) = (v u') (v' u).$$

$$(Q Q^2) = \frac{e^4}{4m^2 c^2 v^2 v'^2 R^2} (2\pi \hbar)^6 \frac{1}{\left(\frac{E'}{c} + mc\right)^2} \left[\begin{aligned} & \frac{v(\alpha M') u' v' (\alpha M')}{H^2 (\alpha M')^2 (v-v')^2 + \frac{M'}{3(M'H)(M'\tilde{H})(v+v')^2}} \\ & + M'^2 \frac{(\alpha H)(\alpha \tilde{H})(v+v')^2}{(u'v'u)} + \frac{4v^2}{\hbar^2} \frac{(\alpha M')(\tilde{\alpha} M')}{v[\alpha M'] [\tilde{\alpha} M']} \\ & + \left(\frac{E'}{c} + mc\right)^2 (v^2 v')^2 [\alpha \tilde{E}] [\tilde{\alpha} \tilde{E}] + \left\{ \frac{\hbar v'}{c} n(n'\tilde{E})(v-v') - \frac{(v+v')^2}{v} mc \tilde{E} \right. \\ & \left. + 2\frac{\hbar v v'}{c} n'(n'\tilde{E}) \right\} \left\{ \frac{\hbar v'}{c} n(n'\tilde{E})(v-v') + \frac{(v+v')^2}{v} mc \tilde{E} + 2\frac{\hbar v v'}{c} n'(n'\tilde{E}) \right\} \\ & + \frac{v}{\hbar} (\alpha M')^u (\alpha H)^u (M'H)^u (v^2 v'^2) + \frac{v}{\hbar} (\alpha \tilde{H})^u (\alpha M')^u (H M')^u (v^2 v'^2) \\ & - \frac{v}{\hbar} (\alpha M')^u (\alpha \tilde{H})^u (H M')^u (v^2 v'^2) - \frac{v}{\hbar} (\alpha H)^u (\alpha M')^u (\tilde{H} M')^u (v^2 v'^2) \\ & + i \frac{2v}{\hbar} (\tilde{\alpha} M')^u \frac{v}{\hbar} (\alpha M')^u [\alpha M']^u (v-v') - 2i \frac{v}{\hbar} (\alpha M')^u ([\alpha M'] \cdot \tilde{H})^u (\alpha M')^u (v-v') \end{aligned} \right]$$

$$\begin{aligned}
& -\left(\frac{E'}{c}+mc\right)(v-v')^2 (\alpha M')^{\alpha'v'} (H \cdot [\alpha \tilde{E}])^{\alpha} - \left(\frac{E'}{c}+mc\right)(v-v')^2 ([\alpha \tilde{E}] \cdot \tilde{H})^{\alpha'v'} (\alpha M')^{\alpha} \\
& - \frac{1}{2} i \hbar (\alpha M') (v-v') (\tilde{X} \tilde{H}) + i \hbar (\alpha M') (v-v') (\tilde{X} \tilde{H}) \\
& - (H M') (v+v')^2 (\alpha M') (\alpha \tilde{H}) - (H \tilde{M}') (v+v')^2 (\alpha H) (\alpha M') \\
& + \frac{2i\gamma}{\hbar} (\tilde{\alpha} M') (H M') (v+v') (\alpha \cdot [\alpha M']) - \frac{2i\gamma}{\hbar} (\alpha M') (M' \tilde{H}) \left([\alpha M'] \alpha \right)^{(v+v')} \\
& - \left(\frac{E'}{c}+mc\right)(v^2-v'^2) (H M') (\alpha \cdot [\alpha \tilde{E}]) - \left(\frac{E'}{c}+mc\right)(v^2-v'^2) (M' \tilde{H}) ([\alpha \tilde{E}] \alpha) \\
& - i (H M') (v+v') (\alpha \tilde{X}) + i (H \tilde{M}') (v+v') (\alpha X) \\
& - \frac{2i\gamma}{\hbar} (\tilde{\alpha} M') (\alpha H) (M' \cdot [\alpha M']) (v+v') + \frac{2i\gamma}{\hbar} (\alpha M') ([\alpha X M'] \cdot M') (\alpha \tilde{H}) \\
& \left(+ i (v+v') (\alpha H) (M' \tilde{X}) - i (v+v') (M' X) (\alpha \tilde{H}) \right. \\
& \left. + \left(\frac{E'}{c}+mc\right)(v^2-v'^2) (\alpha H) (M' \cdot [\alpha \tilde{E}]) + \left(\frac{E'}{c}+mc\right)(v^2-v'^2) (\alpha \tilde{H}) (M' \cdot [\alpha \tilde{E}]) \right) \\
& + \frac{2i\gamma}{\hbar} (\alpha M') \left(\frac{E'}{c}+mc\right)(v-v') ([\alpha M'] \cdot [\alpha \tilde{E}]) - \frac{2i\gamma}{\hbar} (\tilde{\alpha} M') \left(\frac{E'}{c}+mc\right)(v-v') ([\alpha \tilde{E}] \cdot [\alpha M']) \\
& - \frac{2(\alpha M')\gamma}{\hbar} ([\alpha M'] \cdot \tilde{X}) - \frac{2\gamma}{\hbar} (\tilde{\alpha} M') (X \cdot [\alpha M']) \\
& + i \left(\frac{E'}{c}+mc\right)(v-v') ([\alpha \tilde{E}] \cdot \tilde{X}) - i \left(\frac{E'}{c}+mc\right)(v-v') ([\alpha \tilde{E}] X) \\
& + X \tilde{X} \Big]
\end{aligned}$$

$$\begin{aligned}
(\mathcal{O} \tilde{\mathcal{O}}) = & \frac{e^4}{4m^2 c^2 v^2 v'^2} \rho^2 (2\pi\hbar)^6 \frac{1}{(\frac{E'}{c} + mc)^2} \left[\begin{aligned}
& H^2 (v(\alpha M') u) (v'(\alpha M') u) (v-v')^2 \\
& + M'^2 (v+v')^2 (v(\alpha H) u) (v'(\alpha \tilde{H}) u) + \frac{4}{\hbar^2} (\alpha M') (\tilde{\alpha} M') (v[\alpha M'] u) (v'[\alpha M'] u) \\
& + (\frac{E'}{c} + mc)^2 (v-v')^2 (v[\alpha \tilde{E}] u) (v'[\alpha \tilde{E}] u) + (v\chi u') (\tilde{v}\tilde{\chi} u) \\
& + (M'\tilde{H})(v^2 v'^2) (v(\alpha M') u') (v'(\alpha H) u) + (HM')(v^2 v'^2) (v(\alpha \tilde{H}) u) (v'(\alpha M') u) \\
& - (HM')(v^2 v'^2) (v(\alpha M') u') (v'(\alpha \tilde{H}) u) - (\tilde{H}M')(v^2 v'^2) (v(\alpha H) u') (v'(\alpha M') u) \\
& + 2\frac{i}{\hbar} v (\tilde{\alpha} M') (v^2 v') (v(\alpha M') u') (v'(H[\alpha M']) u) - 2\frac{i}{\hbar} v (\alpha M') (v-v') (v[\alpha M'] \tilde{H} u) (v'(\alpha M') u) \\
& - (\frac{E'}{c} + mc)(v-v')^2 (v(\alpha M') u') (v'(\tilde{H}[\alpha \tilde{E}]) u) - (\frac{E'}{c} + mc)(v-v')^2 (v([\alpha \tilde{E}], \tilde{H}) u') (v'(\alpha M') u) \\
& - i(v-v') (v(\alpha M') u') (v'(\tilde{\chi} H) u) + i(v-v') (v(\chi \tilde{H}) u') (v'(\alpha M') u) \\
& - (HM')(v+v')^2 (v(\alpha M') u') (v'(\alpha \tilde{H}) u) - (\tilde{H}M')(v+v')^2 (v(\alpha H) u') (v'(\alpha M') u) \\
& + 2\frac{i}{\hbar} v (\tilde{\alpha} M') (HM')(v+v') (v^2(\alpha) u) (v'[\alpha M'] u) - \frac{2i}{\hbar} v (\alpha M') (\tilde{H}M')(v+v') (v[\alpha M'] u) (v'\alpha u) \\
& - (\frac{E'}{c} + mc)(v^2 v'^2) (HM') (v(\alpha) u') (v'[\alpha \tilde{E}] u) - (\frac{E'}{c} + mc)(v^2 v'^2) (\tilde{H}M') (v[\alpha \tilde{E}] u') (v'\alpha u) \\
& - i(HM')(v+v') (v\alpha u') (v'\tilde{\chi} u) + i(\tilde{H}M')(v+v') (v\tilde{\alpha} u') (v'\alpha u) \\
& - \frac{2i}{\hbar} v (\tilde{\alpha} M') (v+v') (v(\alpha H) u') (v'(\tilde{M}'[\alpha M']) u) + \frac{2i}{\hbar} v (\alpha M') (v+v') (v([\alpha M'] \tilde{M}') u') (v'(\alpha \tilde{H}) u) \\
& + (\frac{E'}{c} + mc)(v^2 v'^2) (v(\alpha H) u') (v'(\tilde{M}'[\alpha \tilde{E}]) u) + (\frac{E'}{c} + mc)(v^2 v'^2) (v'(\tilde{M}'[\alpha \tilde{E}]) u) (v'(\alpha \tilde{H}) u) \\
& + i(v+v') (v(\alpha H) u') (v'(\tilde{M}'\tilde{\chi}) u) - i(v+v') (v(\tilde{M}'\chi) u') (v'(\alpha \tilde{H}) u) \\
& + \frac{2i}{\hbar} v (\alpha M') (\frac{E'}{c} + mc)(v-v') (v[\alpha M'] u') (v'[\alpha \tilde{E}] u) - \frac{2i}{\hbar} v (\tilde{\alpha} M') (\frac{E'}{c} + mc)(v-v') \\
& \quad \quad \quad (v[\alpha \tilde{E}] u') (v'[\alpha M'] u) \\
& - \frac{2(\alpha M')}{\hbar} v (v[\alpha M'] u') (v'\tilde{\chi} u) - \frac{2(\tilde{\alpha} M')}{\hbar} v (v\tilde{\chi} u') (v'[\alpha M'] u) \\
& + i(\frac{E'}{c} + mc)(v-v') (v[\alpha \tilde{E}] u') (v'\tilde{\chi} u) - i(\frac{E'}{c} + mc)(v-v') (v\chi u) (v'[\alpha \tilde{E}] u) \\
& + (v\chi u') (v\tilde{\chi} u) \end{aligned} \right]
\end{aligned}$$

$$(v \sigma_i u')(v' \sigma_i u')$$

$$S \sigma_1 S^{-1} = (b_0^2 + b_1^2 - b_2^2 - b_3^2) \sigma_1 - 2(b_0 b_2 - b_1 b_3) \sigma_3 + 2(b_0 b_3 + b_1 b_2) \sigma_2$$

$$\begin{aligned} S \sigma_2 S^{-1} &= \sigma_2 (b_0 + i b_1 \sigma_1 - i b_2 \sigma_2 + i b_3 \sigma_3) (b_0 + i b_1 \sigma_1 + i b_2 \sigma_2 + i b_3 \sigma_3) \\ &= \sigma_2 (b_0^2 - b_1^2 + b_2^2 - b_3^2 + 2i b_0 b_1 \sigma_1 + 2i b_0 b_3 \sigma_3 - 2i b_1 b_2 \sigma_3 + 2i b_2 b_3 \sigma_1) \\ &= (b_0^2 - b_1^2 + b_2^2 - b_3^2) \sigma_2 + 2(b_0 b_1 + b_2 b_3) \sigma_3 + 2(b_1 b_2 - b_0 b_3) \sigma_1 \end{aligned}$$

$$\begin{aligned} b_0^2 - b_1^2 + b_2^2 - b_3^2 &= \cos^2 \frac{1}{2} \theta \cos^2 \frac{1}{2} (\psi + \phi) - \sin^2 \frac{1}{2} \theta \sin^2 \frac{1}{2} (\psi - \phi) \\ &\quad + \sin^2 \frac{1}{2} \theta \cos^2 \frac{1}{2} (\psi - \phi) - \cos^2 \frac{1}{2} \theta \sin^2 \frac{1}{2} (\psi + \phi) \end{aligned}$$

$$= \cos^2 \frac{1}{2} \theta \cos (\psi + \phi) + \sin^2 \frac{1}{2} \theta \cos (\psi - \phi)$$

$$= \cos \theta \cos \psi \cos \phi - \cos \theta \sin \psi \sin \phi$$

$$\overline{(\quad)}^2 = \frac{3}{8} \sigma_2$$

$$\begin{aligned} 2(b_0 b_1 + b_2 b_3) &= \sin \theta \cos \frac{1}{2} (\psi + \phi) \sin \frac{1}{2} (\psi - \phi) + \sin \theta \cos \frac{1}{2} (\psi - \phi) \sin (\psi + \phi) \\ &= \sin \theta \sin \psi \end{aligned}$$

$$\therefore \overline{2(b_0 b_1 + b_2 b_3)} = \frac{1}{4} \sigma_3$$

$$\begin{aligned} 2(b_1 b_2 - b_0 b_3) &= \sin \theta \left[\cos \frac{1}{2} (\psi + \phi) \sin \frac{1}{2} (\psi - \phi) - \cos \frac{1}{2} (\psi - \phi) \sin (\psi + \phi) \right] \\ &= \sin \theta \left[\sin \psi \cos \phi - \cos \psi \sin \phi - \cos \psi \sin \phi - \sin \psi \cos \phi \right] \\ &= -2 \sin \theta \sin \psi \cos \phi \end{aligned}$$

$$\therefore \overline{2(b_1 b_2 - b_0 b_3)} = \frac{3}{8} \sigma_1$$

$$\therefore \text{total} = 1$$

$$\begin{aligned}
& (S\sigma_1 S^{-1})(S\sigma_2 S^{-1}) \\
&= \left\{ (b_0^2 + b_1^2 - b_2^2 - b_3^2)\sigma_1 - 2(b_0 b_2 - b_1 b_3)\sigma_3 + 2(b_0 b_3 + b_1 b_2)\sigma_2 \right\} \\
&\quad \left\{ (b_0 - b_1^2 + b_2^2 - b_3^2)\sigma_2 + 2(b_0 b_1 + b_2 b_3)\sigma_3 + 2(b_1 b_2 - b_0 b_3)\sigma_1 \right\} \\
&= \left\{ (\cos\theta \cos\varphi \cos\psi - \sin\psi \sin\varphi)\sigma_1 + (\sin\psi \cos\varphi + \cos\theta \cos\psi \sin\varphi)\sigma_2 - \sin\theta \sin\varphi \sigma_3 \right\} \\
&\quad \left\{ (\cos\psi \cos\varphi - \cos\theta \sin\varphi \sin\psi) \right. \\
&\quad \left. (\sin\psi \cos\varphi + \cos\theta \cos\psi \sin\varphi)\sigma_1 + (\cos\psi \cos\varphi - \cos\theta \sin\varphi \sin\psi)\sigma_2 + \sin\theta \sin\varphi \sigma_3 \right\} \\
&\quad \leftarrow -(\cos\theta \sin\psi \cos\varphi + \cos\psi \sin\varphi)
\end{aligned}$$

$$\therefore \overline{(S\sigma_1 S^{-1})(S\sigma_2 S^{-1})} = 0 \cdot \overline{(S\sigma_2 S^{-1})(S\sigma_1 S^{-1})} = 0.$$

$$\begin{aligned}
S\sigma_3 S^{-1} &= \sigma_3 (b_0 + i b_1 \sigma_1 + i b_2 \sigma_2 - i b_3 \sigma_3) (b_0 + i b_1 \sigma_1 + i b_2 \sigma_2 + i b_3 \sigma_3) \\
&= \sigma_3 (b_0^2 - b_1^2 - b_2^2 + b_3^2 + 2i b_0 b_1 \sigma_1 + 2i b_0 b_2 \sigma_2 + 2i b_1 b_3 \sigma_2 - 2i b_2 b_3 \sigma_1) \\
&= (b_0^2 - b_1^2 - b_2^2 + b_3^2)\sigma_3 + 2(b_2 b_3 - b_0 b_1)\sigma_2 + 2(b_0 b_2 + b_1 b_3)\sigma_1
\end{aligned}$$

$$b_0^2 - b_1^2 - b_2^2 + b_3^2 = \cos^2 \frac{1}{2} \theta - \sin^2 \frac{1}{2} \theta = \cos \theta. \quad \therefore (\quad)^2 = \frac{1}{2} \quad \sigma_3$$

$$\begin{aligned}
2(b_2 b_3 - b_0 b_1) &= \sin \theta \left\{ \sin \frac{1}{2} (\psi + \varphi) \cos \frac{1}{2} (\psi - \varphi) - \cos \frac{1}{2} (\psi + \varphi) \sin \frac{1}{2} (\psi - \varphi) \right\} \\
&= \sin \theta \sin \varphi. \quad (\quad)^2 = \frac{1}{4} \quad \sigma_2
\end{aligned}$$

$$\begin{aligned}
2(b_0 b_2 + b_1 b_3) &= \sin \theta \left\{ \cos \frac{1}{2} (\psi + \varphi) \cos \frac{1}{2} (\psi - \varphi) + \sin \frac{1}{2} (\psi + \varphi) \sin \frac{1}{2} (\psi - \varphi) \right\} \\
&= \sin \theta \cos \varphi. \quad (\quad)^2 = \frac{1}{4} \quad \sigma_1
\end{aligned}$$

$$\begin{aligned}
& (S\sigma_2 S^{-1})(S\sigma_3 S^{-1}) \\
&= \left\{ -(\cos\theta \sin\psi \cos\varphi + \cos\psi \sin\varphi)\sigma_1 + (\cos\psi \cos\varphi - \cos\theta \sin\varphi \sin\psi)\sigma_2 \right. \\
&\quad \left. + \sin\theta \sin\varphi \sigma_3 \right\} \\
&\quad \times \left\{ \sin\theta \cos\varphi \sigma_1 + \sin\theta \sin\varphi \sigma_2 + \cos\theta \sigma_3 \right\}
\end{aligned}$$

$$\therefore \overline{(S \sigma_3 S^{-1})(S \sigma_2 S^{-1})} = \overline{(S \sigma_2 S^{-1})(S \sigma_3 S^{-1})} = 0$$

$$\therefore \overline{\{v(\sigma M') u'\} \{v'(\sigma M') u\}}$$

$$= \overline{\{v(\sigma_1 M_1' + \sigma_2 M_2' + \sigma_3 M_3') u'\} \{v'(\sigma_1 M_1' + \sigma_2 M_2' + \sigma_3 M_3') u\}}$$

$$= (v(\sigma_1 M_1') u') (v'(\sigma_1 M_1') u) + (v(\sigma_2 M_2') u') (v'(\sigma_2 M_2') u) + (v(\sigma_3 M_3') u') (v'(\sigma_3 M_3') u)$$

$$= \cancel{\sigma_1} \left(\overline{\sigma M'} \cdot \widetilde{\sigma M'} \right)$$

$$v'(\sigma_3 M_3') u)$$

$$= \cancel{M_1'^2 + M_2'^2 + M_3'^2} \{ (v u') (v' u) \} = \underline{M'^2 (v u') (v' u)}$$

$$= \cancel{M'} M'^2 (v u') (v' u)$$

$$(v \sigma u') (v' \sigma u) = (v u') (v' u)$$

$$v \alpha u' = s \quad v' \alpha u = \tilde{s}$$

Voraussetzung $S_1 \tilde{S}_1 = (v u')_{(v' u)} S_1 \tilde{S}_2 = \tilde{S}_2 \tilde{S}_1 = \dots = 0$

$$(S \tilde{S}) = 3 (v u')_{(v' u)}$$

$$(A S)(B \tilde{S}) = (A B)_{(v u')_{(v' u)}}$$

$$H^2 (v (\alpha M') u') (v' (\alpha M') u) (v - v')^2$$

$$([A S A][\tilde{S} B]) = (S \tilde{S})(A B) - (\tilde{S} A)(S B)$$

$$= \{3(A B) - A B\}_{(v u')_{(v' u)}}$$

$$= 2(A B)_{(v u')_{(v' u)}}$$

$$= H^2 \cancel{(\alpha M') (\tilde{\alpha} M')} = H^2 \{ (S M') (\tilde{S} M') \} = H^2 M'^2 (v u')_{(v' u)} \frac{1}{(v - v')^2}$$

$$(v \alpha u') (v' \alpha u) (H M') (\tilde{H} M') (v + v')^2$$

$$(S [\tilde{S} M']) = ([\tilde{S} \tilde{S}] M')_{\tilde{S} \tilde{S}}$$

$$= 3 (H M') (\tilde{H} M') (v + v')^2 (v u')_{(v' u)}$$

$$M'^2 (v + v')^2 (v (\alpha H) u') (v' (\alpha \tilde{H}) u) = M'^2 (v + v')^2 H^2 (v u')_{(v' u)}$$

$$\frac{4v^2}{h^2} (a M') (\tilde{a} M') (v [\alpha M'] u') (v' [\alpha M'] u) = \frac{4v^2}{h^2} (a M') (\tilde{a} M') M'^2 (v u')_{(v' u)}$$

$$(\frac{E'}{c} + mc)^2 (v - v')^2 (v [\alpha \tilde{E}] u') (v' [\alpha \tilde{E}] u) = 2 (\frac{E'}{c} + mc)^2 (v - v')^2 |\tilde{E}|^2$$

$$(v \chi u') (v' \tilde{\chi} u) = |\chi|^2 (v u')_{(v' u)}$$

$$(v (\alpha M') u') (v' (H [\alpha M'] u))$$

$$= (S M') (H [\tilde{S} M']) = (S M') (\tilde{S} [\tilde{M}' H])$$

$$= (M' [\tilde{M}' H])_{(v u')_{(v' u)}}$$

$$= (H [M' \chi M']) \dots = 0$$

$$\frac{2iv}{h} (\tilde{a} M') (H M') (v + v')^2 (v (\alpha M') u') (v' (\alpha M') u)$$

$$- (H M') (v + v')^2 (S (\alpha M') u') (v' (\alpha \tilde{H}) u) - (\tilde{H} M') (v + v')^2 (v (\alpha H) u') (v' (\alpha M') u)$$

$$= \{ - (H M') (\tilde{H} M') - (\tilde{H} M') (H M') \} (v + v')^2 = -2 (H M') (\tilde{H} M') (v + v')^2 (v u')_{(v' u)}$$

$$+ \frac{4iv}{h} (a M') (\frac{E'}{c} + mc) (v - v') (\tilde{E} M') - \frac{4iv}{h} (\frac{E'}{c} + mc) (v - v') (\tilde{E} M')$$

$$= + \frac{4iv}{h} (\frac{E'}{c} + mc) (v - v') \{ (a M') (\tilde{E} M') - (\tilde{a} M') (\tilde{E} M') \} (v u')_{(v' u)}$$

$$\therefore (\mathcal{O} \tilde{\mathcal{O}}) = \frac{e^4}{4m^2 c^4 v v'^2 R^2} (2\pi h)^6 \frac{1}{(\frac{E'}{c} + mc)^2} \left[H^2 M'^2 (v - v')^2 + (H M') (\tilde{H} M') (v + v')^2 + \right. \\ \left. + H^2 M'^2 (v + v')^2 + \frac{4v^2}{h^2} (a M') (\tilde{a} M') M'^2 + 2 (\frac{E'}{c} + mc)^2 (v - v')^2 |\tilde{E}|^2 + |\chi|^2 \right. \\ \left. - \frac{8iv}{h} (\frac{E'}{c} + mc) (v - v') (\tilde{E} M') (\tilde{E} M') \right] (v u')_{(v' u)}$$

$$[n' \chi \mathcal{O}_L] = \frac{e^2}{R} \frac{1}{2mc\gamma\gamma'} \frac{(2\pi\hbar)^3}{(\frac{E'}{c} + mc)} V \left[[n'H](\alpha M')(\gamma - \gamma') + [n'\alpha](HM')(\gamma + \gamma') - \right. \\ \left. - [n'M'](\alpha H)(\gamma + \gamma') - 2\frac{i\gamma}{\hbar}(\alpha M') [n'[\alpha M']] - (\frac{E'}{c} + mc)(\gamma - \gamma') [n'[\alpha \tilde{\mathcal{E}}]] + i[n'\chi] \right] u'$$

$$[n' \chi \tilde{\mathcal{O}}_L] = \frac{e^2(2\pi\hbar)^3}{2mcR\gamma\gamma'(\frac{E'}{c} + mc)} V' \left[[n'\tilde{H}](\alpha M')(\gamma - \gamma') + [n'\alpha](\tilde{H}M')(\gamma + \gamma') - \right. \\ \left. - [n'M'](\alpha \tilde{H})(\gamma + \gamma') + 2\frac{i\gamma}{\hbar}(\alpha M') [n'[\alpha M']] - (\frac{E'}{c} + mc)(\gamma - \gamma') [n'[\alpha \tilde{\mathcal{E}}]] - i[n'\tilde{\chi}] \right] u$$

$$\overline{[n' \chi \mathcal{O}_L][n' \chi \tilde{\mathcal{O}}_L]} = \frac{e^4(2\pi\hbar)^6}{4m^2c^2R^2\gamma^2\gamma'^2(\frac{E'}{c} + mc)^2} V \left[\right] u' V' \left[\right] u \\ = \frac{e^4(2\pi\hbar)^6}{4m^2c^2R^2\gamma^2\gamma'^2(\frac{E'}{c} + mc)^2} \left[\begin{aligned} & \{ |H|^2 (n'H)(n'\tilde{H}) \} M'^2 (\gamma - \gamma')^2 + 2(HM')(\tilde{H}M')(\gamma + \gamma')^2 + \\ & + \{ M'^2 (n'M')^2 \} |H|^2 (\gamma + \gamma')^2 + \frac{4\gamma^2}{\hbar^2} (\alpha M')(\tilde{\alpha} M') (\cancel{(\alpha M')^2}) \{ M'^2 + (n'M')^2 \} + \cancel{|H|^2 (n'H)(n'\tilde{H})} \\ & + (\frac{E'}{c} + mc)^2 (\gamma - \gamma')^2 \{ |\mathcal{E}|^2 + (n'\mathcal{E})(n'\tilde{\mathcal{E}}) \} + |\chi|^2 (n'\chi)(n'\tilde{\chi}) + \\ & + \{ (HM')(\tilde{H}M') - (n'H)(n'M')(\tilde{H}M') \} (\gamma^2 - \gamma'^2) - \{ (HM')(\tilde{H}M') \tilde{\sigma} (n'H)(n'M')(\tilde{H}M') \} (\gamma^2 - \gamma'^2) \\ & + \{ (\tilde{H}M')(HM') - (n'\tilde{H})(n'M')(HM') \} (\gamma^2 - \gamma'^2) - \{ (\tilde{H}M')(HM') - (n'\tilde{H})(n'M')(HM') \} (\gamma^2 - \gamma'^2) \\ & + (\frac{E'}{c} + mc)(\gamma - \gamma')^2 (n'H)(n'[\mathcal{M}'\tilde{\mathcal{E}}]) + (\frac{E'}{c} + mc)(\gamma - \gamma')^2 (n'\tilde{H})(n'[\mathcal{M}'\tilde{\mathcal{E}}]) \\ & - \{ (HM')(\tilde{H}M') - (n'\tilde{H})(n'M')(HM') \} (\gamma + \gamma')^2 - \{ (\tilde{H}M')(HM') - (n'H)(n'M')(\tilde{H}M') \} (\gamma + \gamma')^2 \\ & + 2\frac{i\gamma}{\hbar}(\tilde{\alpha} M')(n'M')(n'[\mathcal{M}'H]) - 2\frac{i\gamma}{\hbar}(\alpha M')(n'M')(n'[\mathcal{M}'\tilde{H}]) \\ & - (\frac{E'}{c} + mc)(n'M')(n'[\mathcal{H}\tilde{\mathcal{E}}])(\gamma^2 - \gamma'^2) - (\frac{E'}{c} + mc)(n'M')(n'[\tilde{H}\mathcal{E}])(\gamma^2 - \gamma'^2) \\ & + 2\frac{i\gamma}{\hbar}(\gamma - \gamma')(\frac{E'}{c} + mc)(\alpha M') \{ (\mathcal{M}'\tilde{\mathcal{E}}) + (n'M')(n'\tilde{\mathcal{E}}) \} \\ & - 2\frac{i\gamma}{\hbar}(\gamma - \gamma')(\frac{E'}{c} + mc)(\tilde{\alpha} M') \{ (\mathcal{M}'\mathcal{E}) + (n'M')(n'\mathcal{E}) \} \end{aligned} \right]$$

$$(\frac{E'}{c} + mc)^2 (\gamma - \gamma')^2 \left([n'H] [\tilde{n}'\tilde{H}] \right) (\alpha M')^2 (\gamma - \gamma')^2$$

$$= \left\{ |H|^2 - (n'H)(n'\tilde{H}) \right\} M'^2 (\gamma - \gamma')^2$$

$$[n'\alpha] [\tilde{n}'\alpha] (HM') (\tilde{H}M') (\gamma + \gamma')^2 = \left\{ (\alpha)^2 - (n'\alpha)(n'\alpha) \right\} (HM') (\tilde{H}M') (\gamma + \gamma')^2$$

$$= 3 \frac{4\gamma^2}{h^2} (HM') (\tilde{H}M') (\gamma + \gamma')^2$$

$$[n'M'] [\tilde{n}'M'] (\alpha H) (\alpha \tilde{H}) (\gamma + \gamma')^2 = \left\{ M'^2 - (n'M')^2 \right\} |H|^2 (\gamma + \gamma')^2$$

$$= \frac{4\gamma^2}{h^2} (\alpha M') (\tilde{\alpha} M') [n'[\alpha M']] [\tilde{n}'[\alpha M']] = \frac{4\gamma^2}{h^2} (\alpha M') (\tilde{\alpha} M')$$

$$- (n'[\alpha M']) (\tilde{n}'[\alpha M']) \left\{ = \frac{4\gamma^2}{h^2} (\alpha M') (\tilde{\alpha} M') \left\{ M'^2 - (\alpha[M'n']) (\alpha[M'n']) \right\} \right.$$

$$= \frac{4\gamma^2}{h^2} (\alpha M') (\tilde{\alpha} M') \left\{ M'^2 - [M'n'] [M'n'] \right\} = \frac{4\gamma^2}{h^2} (\alpha M') (\tilde{\alpha} M') \left\{ M'^2 - M'^2 + (n'M')^2 \right\}$$

$$(\frac{E'}{c} + mc)^2 (\gamma - \gamma')^2 [n'[\alpha \tilde{E}]] [\tilde{n}'[\alpha \tilde{E}]] = (\alpha [\tilde{E}n']) (\alpha [\tilde{E}n'])$$

$$= \frac{4\gamma^2}{h^2} (\alpha M') (\tilde{\alpha} M') \left\{ (\alpha M') (\tilde{\alpha} M') - (n'[\alpha M']) (\tilde{n}'[\alpha M']) \right\}$$

$$= \frac{4\gamma^2}{h^2} (\alpha M') (\tilde{\alpha} M') \left\{ (\alpha)^2 M'^2 - (\alpha M') (\alpha M') - (\alpha[M'n']) (\alpha[M'n']) \right\}$$

$$= \dots \left\{ 3 M'^2 - M'^2 - ([M'n']) ([M'n']) \right\}$$

$$= \dots \left\{ 2 M'^2 - M'^2 + (n'M')^2 \right\} = \dots \left\{ M'^2 + (n'M')^2 \right\}$$

$$([n'\chi]) ([\tilde{n}'\tilde{\chi}]) = |\chi|^2 - (n'\tilde{\chi})(n'\chi)$$

$$(\frac{E'}{c} + mc)^2 (\gamma - \gamma')^2 \left([n'[\alpha \tilde{E}]] [\tilde{n}'[\alpha \tilde{E}]] \right)$$

$$= ([\alpha \tilde{E}]) ([\tilde{\alpha} \tilde{E}]) - (n'[\alpha \tilde{E}]) (\tilde{n}'[\alpha \tilde{E}])$$

$$= \dots \alpha^2 |\tilde{E}|^2 - (\alpha \tilde{E})(\alpha \tilde{E}) - (\alpha [\tilde{E}n']) (\alpha [\tilde{E}n'])$$

$$= \dots 3 |\tilde{E}|^2 - |\tilde{E}|^2 - |\tilde{E}|^2 + (n'\tilde{E})(n'\tilde{E})$$

$$2 |\tilde{E}|^2 + (n'\tilde{E})(n'\tilde{E})$$

$$\begin{aligned} & \frac{[n'H][n'a]}{(\alpha M')} (\alpha M') (\tilde{H} M') (v^2 - v'^2) = \left\{ (\alpha H) - (n'H)(n'a) \right\} (\tilde{H} M') (v^2 - v'^2) \quad 16'' \\ & (\alpha M') = \left\{ (H M') - (n'H)(n'M') \right\} (\tilde{H} M') (v^2 - v'^2) \end{aligned}$$

$$\begin{aligned} & - \frac{[n'H][n'M']}{(\alpha M')} (\alpha M') (\alpha \tilde{H}) = - \left\{ (H M') - (n'H)(n'M') \right\} (v^2 - v'^2) (\tilde{H} M') \\ & = \left\{ - (H M') (\tilde{H} M') + (n'M')(n'H) (\tilde{H} M') \right\} (v^2 - v'^2). \end{aligned}$$

$$\begin{aligned} & \frac{2iv}{h} (\alpha M') (v - v') (\alpha M') \left\{ [n'H][n'[\alpha M']] \right\} = \dots (H[\alpha M']) - (n'H)(n'[\alpha M']) \\ & = \dots (\alpha M') (\alpha [M'H]) - (n'H)(\alpha M') (\alpha [M'n']) \\ & = \dots (M'[M'H]) - nH(M'[M'n']) = 0. \end{aligned}$$

$$\begin{aligned} & ([n'\tilde{H}][n'a]) (\alpha M') (H M') (v^2 - v'^2) = \left\{ (\alpha \tilde{H}) - (n'\tilde{H})(n'a) \right\} (\alpha M') (H M') (v^2 - v'^2) \\ & = \left\{ (\tilde{H} M') - (n'\tilde{H})(n'M') \right\} (H M') (v^2 - v'^2). \end{aligned}$$

$$- [n'M'] [n'\tilde{H}] (\alpha H) (\alpha M') (v^2 - v'^2) = - \left\{ (M'\tilde{H}) - (n'\tilde{H})(n'M') \right\} (H M') -$$

$$\begin{aligned} & - \left(\frac{E'}{c} + mc \right) (v - v')^2 (\alpha M') \left\{ [n'H][n'[\alpha \tilde{E}]] \right\} = \dots (H[\alpha \tilde{E}]) - (n'H)(n'[\alpha \tilde{E}]) \\ & = \dots (M'[\tilde{E}H]) - (n'H)(M'[\tilde{E}n']) = \left(\frac{E'}{c} + mc \right) (v^2 - v'^2) (n'H)(n'[M'\tilde{E}]) \\ & = 0 - (n'H) \left\{ \left(\frac{h\nu}{c} n - \frac{h\nu'}{c} n' \right) \cdot [\tilde{E} n'] \right\}. \end{aligned}$$

$$- \left(\frac{E'}{c} + mc \right) (v - v')^2 [n'\tilde{H}] [n'[\alpha \tilde{E}]] (\alpha M')$$

$$= \dots \left\{ (\tilde{H}[\alpha \tilde{E}]) - (n'\tilde{H})(n'[\alpha \tilde{E}]) \right\} (\alpha M')$$

$$= \dots \left\{ 0 - (n'\tilde{H})(M'[\tilde{E}n']) \right. \\ \left. n'[M'\tilde{E}] \right\}$$

$$(2-3') \quad - (\cancel{\alpha} H M') ([n' \alpha] [n' M']) (\cancel{\alpha} \tilde{H}) (\gamma + \gamma')^2 = - (H M') \{ (\alpha M') - (n' \alpha) (n' M') \} \dots 16'''$$

$$= - (H M') \{ (M' \tilde{H}) - (n' \tilde{H}) (n' M') \} (\gamma + \gamma')^2$$

$$= - \{ (H M') (\tilde{H} M') - (n' \tilde{H}) (n' M') (H M') \} (\gamma + \gamma')^2$$

$$(2-3'') \quad - (\alpha H) ([n' M'] [n' \alpha]) (\tilde{H} M') (\gamma + \gamma')^2$$

$$= - (\alpha H) \{ (\alpha M') - (n' \alpha) (n' M') \} \dots = - \{ (H M') (\tilde{H} M') + (n' H) (n' M') (\tilde{H} M') \}$$

$$(2-4')$$

$$\frac{2i\gamma}{\hbar} (H M') (\gamma + \gamma') ([\alpha M'] [\alpha [M' n]]) = \dots \{ 3 M' [M' n] - \alpha M' [M' n] \} = 0$$

$$(2-5')$$

$$- (\frac{E'}{c} + mc) (H M') (\gamma^2 - \gamma'^2) \left| [\alpha n'] [\alpha [\tilde{E} n']] \right| = \dots 3(n' [\tilde{E} n']) - n' [\tilde{E} n'] = 0$$

$$(3-4')$$

$$- \frac{2i\gamma}{\hbar} (\tilde{\alpha} M') (\alpha H) (\gamma + \gamma') ([n' M'] [n' [\alpha M']]) = - \frac{2i\gamma}{\hbar} (\tilde{\alpha} M') (\gamma + \gamma') (\alpha H) \left(\underset{\parallel}{\underset{0}{M' [\alpha M']}} \right)$$

$$- (n' M') (n' [\alpha M']) = + \frac{2i\gamma}{\hbar} (\tilde{\alpha} M') (\gamma + \gamma') (n' M') (H [n' M'])$$

$$(3-4)$$

$$(3-5') \quad (\frac{E'}{c} + mc) (\gamma^2 - \gamma'^2) (\alpha H) [n' M'] [n' [\alpha \tilde{E}]] \quad \alpha [\tilde{E} n']$$

$$= \dots \{ M' [\alpha \tilde{E}] - (n' M') (n' [\alpha \tilde{E}]) \}$$

$$= \dots M' (H \tilde{E}) - (n' M') (H [\tilde{E} n'])$$

$$= - (\frac{E'}{c} + mc) (\gamma^2 - \gamma'^2) (n' M') (n' [H \tilde{E}])$$

$$(5-3') = (\frac{E'}{c} + mc) (\gamma^2 - \gamma'^2) [n' [\alpha \tilde{E}]] [n' M'] (\alpha \tilde{H})$$

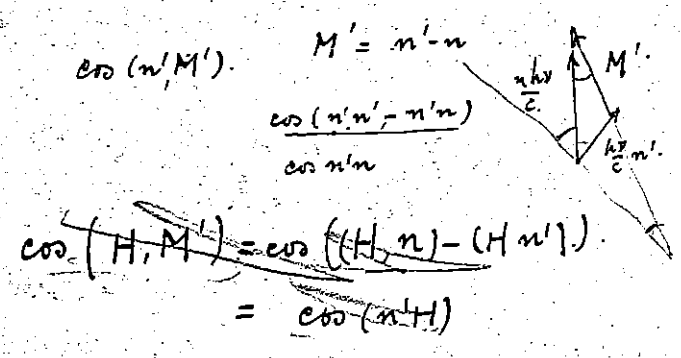
$$= \dots (M' [\alpha \tilde{E}]) - (n' M') (n' [\alpha \tilde{E}]) \quad \alpha [\tilde{E} n'] (\alpha \tilde{H}) = - \dots (n' M') (H [\tilde{E} n'])$$

$$\begin{aligned}
 (4-5) \quad & \frac{2i\gamma}{\hbar} (aM') \left(\frac{E'}{c} + mc \right) (\gamma - \gamma') [n' [aM']] [n' [a\tilde{E}]] \\
 = & \left\{ [aM'] [a\tilde{E}] - (n' [aM']) (n' [a\tilde{E}]) \right. \\
 & \left. (a^2(M'\tilde{E}) - (M'\tilde{E}) - [M'n'] [\tilde{E}'n']) \right. \\
 & 2(M'\tilde{E}) - M'^2 + (n'M')^2 \\
 & (M'\tilde{E}) + (n'M')(n'\tilde{E}) \\
 & (M'\tilde{E}) + (n'M')(n'\tilde{E})
 \end{aligned}$$

(4'-5) -

$$\begin{aligned}
 & \{ |H|^2 (n'H)(n'\tilde{H}) \} M'^2 = |H|^2 \sin^2(n'H) \left(\frac{E'^2}{c^2} - m^2 c^2 \right) \\
 & 2(HM')(H\tilde{M}') = 2 \left(H, \left(\frac{\hbar\gamma n}{c} - \frac{\hbar\gamma' n'}{c} \right) (H, \dots) \right) = 2 |H|^2 \left(\frac{\hbar\gamma'}{c} \right)^2 \cos^2(n'H) \\
 & = 2 |H|^2 |M'|^2 \cos^2(H, \tilde{M}').
 \end{aligned}$$

$$\{ M'^2 (n'M')^2 \} |H|^2 = H^2$$



$$\frac{\hbar\gamma}{c} \left(\frac{\hbar\gamma}{c} - \frac{\hbar\gamma'}{c} (nn') \right)$$

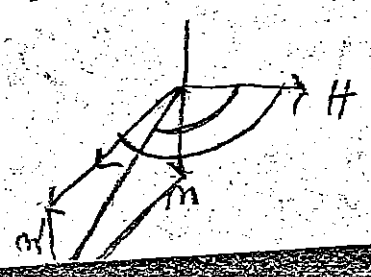
$$\begin{aligned}
 M'^2 &= \left(\frac{\hbar\gamma'}{c} \right)^2 \\
 M'^2 &= \left(\frac{\hbar\gamma}{c} - \frac{\hbar\gamma'}{c} (nn') \right)^2 = \left(\frac{\hbar\gamma'}{c} \right)^2 + \left(\frac{\hbar\gamma}{c} \right)^2 - 2 \left(\frac{\hbar}{c} \right)^2 \gamma\gamma' (nn') \\
 &= \left(\frac{\hbar\gamma'}{c} \right)^2 + \left(\frac{\hbar}{c} \right)^2 \left\{ \gamma^2 - 2\gamma\gamma' (nn') \right\}
 \end{aligned}$$

$$(HM') = |H| |M'| \cos(HM') = |H| |M'| \cos(n'H)$$

$$(HM')(H\tilde{M}') = |H|^2 M'^2 \cos^2(HM')$$

$$(H, M') = |H| |M'| \cos(HM') = - \frac{\hbar\gamma'}{c} (H, n') = - \frac{\hbar\gamma'}{c} |H| \cos(H, n')$$

$$|M'| \cos(H, M') = - \frac{\hbar\gamma'}{c} \cos(H, n')$$



$$\left\{ \frac{h\nu'}{c} n(n'\epsilon)(\nu-\nu') + mc \epsilon \frac{(\nu+\nu')^2}{\nu} \right\} \left\{ \frac{h\nu'}{c} n(n'\tilde{\epsilon})(\nu-\nu') + mc \tilde{\epsilon} \frac{(\nu+\nu')^2}{\nu} \right\}$$

$$= \frac{h^2 \nu'^2}{c^2} (\nu-\nu')^2 (n'\epsilon)(n'\tilde{\epsilon}) + m^2 c^2 |\epsilon|^2 \frac{(\nu+\nu')^4}{\nu^2}$$

$$= \frac{h^2 \nu'^2}{c^2} (\nu-\nu')^2 |\epsilon|^2 \cos^2(n'\epsilon) + m^2 c^2 |\epsilon|^2 \frac{(\nu+\nu')^4}{\nu^2}$$

$$\left\{ \frac{h\nu'}{c} (nn')(n'\epsilon)(\nu-\nu') + mc (n'\epsilon) \frac{(\nu+\nu')^2}{\nu} \right\} \left\{ \frac{h\nu'}{c} (nn')(n'\tilde{\epsilon})(\nu-\nu') + mc (n'\tilde{\epsilon}) \frac{(\nu+\nu')^2}{\nu} \right\}$$

$$= \frac{h^2 \nu'^2}{c^2} |\epsilon|^2 \cos^2(nn') \cos^2(n'\epsilon) (\nu-\nu')^2 + m^2 c^2 |\epsilon|^2 \frac{(\nu+\nu')^4}{\nu^2} \sin^2(n'\epsilon)$$

$$+ 2 \frac{h\nu'}{c} m (\nu+\nu')^2 (\nu-\nu') |\epsilon|^2 \cos^2(n'\epsilon)$$

$$\nu' = \frac{\nu}{1 + \frac{h\nu}{mc\nu} (1 - (nn'))}$$

$$\frac{\nu}{\nu'} = 1 + \frac{h\nu}{mc\nu} (1 - (nn'))$$

$$\frac{\nu-\nu'}{\nu'} = \frac{h\nu}{mc\nu} (1 - (nn')) \left\{ \frac{1}{\left(1 + \frac{h\nu}{mc\nu} (1 - (nn'))\right)} + 1 \right\}$$

$$\frac{(\nu+\nu')}{\nu} = 2 + \frac{h\nu}{mc\nu} (1 - (nn'))$$

$$\frac{h^2 \nu'^2 (\nu-\nu')^2}{c^2} |\epsilon|^2 \cos^2(n'\epsilon) \sin^2(n'n)$$

$$+ m^2 c^2 \frac{(\nu+\nu')^4}{\nu^2} |\epsilon|^2 \sin^2(n'\epsilon)$$

$$- 2 \frac{h\nu}{c} \frac{\nu'}{\nu} (\nu+\nu')^2 (\nu-\nu') |\epsilon|^2 \cos^2(n'\epsilon)$$

$$j = -(j+1)$$

$$-(j+1) = j$$

$$j = -1$$

$$j = -1$$

$$j = -1$$

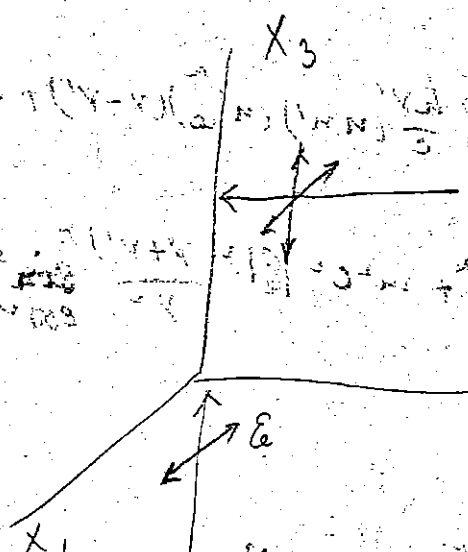
$$j = -1$$

$$j = -1$$

$$\phi_0 \phi_2 \psi_0' = i \phi_0 \phi_1 \psi_0'$$

$$\phi_0 \phi_1 \psi_0' = -i \phi_0 \phi_2 \psi_0'$$

$$\phi_0 \phi_3 \psi_0' = -\phi_0 \psi_0'$$



$$E = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$K = \gamma - \gamma' + \frac{2mc^2}{h} = \frac{\gamma^2 + \gamma'^2 - 2\gamma\gamma'(nn')}{\gamma - \gamma'}$$

$$\omega^2 = (\gamma - \gamma')K \quad \omega = n\gamma - n'\gamma'$$

$$(n'\omega) = (nn')\gamma - \gamma' = (\gamma - \gamma')\left(1 - \frac{mc^2}{h\gamma'}\right)$$

$$(n\omega) = \gamma - (nn')\gamma' = (\gamma - \gamma')\left(1 + \frac{mc^2}{h\gamma}\right)$$

$$\frac{mc^2}{h} = \frac{\gamma\gamma'}{\gamma - \gamma'} (1 - (nn'))$$

$$(nn')\gamma = \gamma - \frac{\gamma - \gamma'}{\gamma'} \frac{mc^2}{h}$$

$$(nn')\gamma' = \gamma' - \frac{\gamma - \gamma'}{\gamma} \frac{mc^2}{h}$$

$$(nn') = 1 - \frac{\gamma - \gamma'}{\gamma\gamma'} \frac{mc^2}{h}$$

$$1 - (nn') = \frac{\gamma - \gamma'}{\gamma\gamma'} \frac{mc^2}{h}$$

$$1 - (nn')^2 = \frac{2(\gamma - \gamma')}{\gamma\gamma'} \frac{mc^2}{h} - \frac{(\gamma - \gamma')^2 m^2 c^4}{\gamma^2 \gamma'^2 h^2}$$

$$(n'[n\epsilon])^2 = \epsilon^2(1 - (nn')^2) - (n'\epsilon)^2$$

$$\frac{\gamma'}{\gamma - \gamma'} (1 - (nn')) = \frac{1}{\alpha}$$

$$\alpha = \frac{h\gamma}{mc^2}$$

$$\beta = \frac{h\gamma}{mc^2} (1 - (nn')) = \alpha (1 - (nn'))$$

$$\frac{mc^2}{h} = \frac{\gamma}{\alpha}$$

$$\gamma' = \frac{\gamma}{1 + \alpha(1 - (nn'))} = \frac{\gamma}{1 + \beta}$$

$$\gamma - \gamma' = \beta\gamma' = \frac{\beta\gamma}{1 + \beta}$$

$$\gamma + \gamma' = \gamma \left(\frac{2 + \beta}{1 + \beta} \right)$$

$$\gamma\gamma' = \frac{\gamma^2}{1 + \beta}$$

$$(nn') = 1 - \frac{\beta}{\alpha}$$

$$1 - (nn')^2 = \frac{\beta}{\alpha} \left(2 - \frac{\beta}{\alpha} \right)$$

$$\therefore 1 - (nn') = \frac{\beta}{\alpha}$$

$$1 + (nn') = 2 - \frac{\beta}{\alpha}$$

$$K = \frac{\gamma(2 + 2\beta + \alpha\beta)}{\alpha(1 + \beta)}$$

$$f = \frac{1}{2(2\pi h)} \left(1 + \frac{mc^2}{E'} \right)$$

$$\frac{E'}{mc^2} \frac{\gamma'}{\gamma} \text{ factor}$$

$$\frac{1}{\gamma'} - \frac{1}{\gamma} = \frac{\beta}{\gamma}$$

$$\frac{1}{\gamma'} + \frac{1}{\gamma} = \frac{1}{\gamma} (2 + \beta)$$

$$[S\tilde{S}] = 2i\gamma n$$

$$(AS)(B\tilde{S}) = \gamma \{ (AB) + i(n[AB]) \}$$

$$(AS)\bar{d} = (An)\gamma$$

$$\bar{d}(B\tilde{S}) = (Bn)\gamma$$

$$(S\tilde{S}) = 3\gamma$$

$$\bar{S}\bar{d} = \bar{d}\tilde{S} = \gamma n$$

$$d\bar{d} = \gamma$$

$$\frac{\alpha(1 + \beta)}{\gamma(2 + 2\beta + \alpha\beta)}$$

$$\left(1 + \frac{1}{1 + \beta} \right)^3$$

$$\frac{1}{\alpha} = \frac{\gamma'}{\gamma - \gamma'} (1 - (nn'))$$

$$= \frac{1}{\frac{\gamma}{\gamma'} - 1} (1 - (nn')) = \frac{1}{\beta} (1 - (nn'))$$

4

8

$$H' = \frac{(2\pi\hbar)^3 e^2 \nu'}{2mc^2(\nu - \nu' + \frac{2mc^2}{\hbar})^2} \left\{ d\left(\frac{1}{\nu}(n'\epsilon)(\nu' - \nu)[n'n] - \nu'(\frac{1}{\nu} + \frac{1}{\nu'})^2 \frac{mc^2}{\hbar} [n'\epsilon]\right) \right.$$

$$- i \left(\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) \left[(S, n\nu - n'\nu') \left((n'\epsilon)n - (n'n)\epsilon \right) + \left(\nu - \nu' + \frac{2mc^2}{\hbar}\right) \left((Sn')\epsilon - (n'\epsilon)S \right) \right] \right.$$

$$+ \frac{2}{\nu} (\epsilon n') \left((n'S)(n\nu - n'\nu') + (\nu' - (nn')\nu)S \right) - \left(\frac{1}{\nu'} + \frac{1}{\nu}\right) \left((n[\epsilon S]) \frac{mc^2}{\hbar} [n'n] \right.$$

$$\left. \left. + \nu'(n'[n\epsilon])[n'S] \right) \right\} e^{i\nu'(t - \frac{r}{c})} + \dots$$

$$A' = \frac{e}{\hbar} (2\pi\hbar)^3 \frac{e}{2mc\gamma\gamma'} \left[\left\{ H(SM')(\gamma-\gamma') + S(HM')(\gamma+\gamma') - M'(SH)(\gamma+\gamma') \right. \right. \\ \left. \left. - 2i \frac{(aM')\gamma}{\hbar} [S \times M'] - \left(\frac{E'}{c} + mc \right) (\gamma-\gamma') [S \times \mathcal{E}] \right\} \right. \\ \left. + i d \left\{ [H \times M'](\gamma+\gamma') + 2c \frac{(E M')M'}{\hbar} - \left(\frac{E'}{c} + mc \right) \mathcal{E}(\gamma+\gamma') \right\} \right].$$

$$\cdot \frac{1}{\frac{E'}{c} + mc} \quad \text{wobei} \quad S = v \sim u' \quad d = v u'$$

$$= \frac{e^2}{\hbar} (2\pi\hbar)^3 \frac{1}{2mc\gamma\gamma' (2mc + \frac{\hbar}{c}(\gamma-\gamma'))} \frac{\hbar}{c} \left[(\gamma-\gamma') \left\{ (n \times \mathcal{E})(S, n\gamma - n'\gamma') \right. \right. \\ \left. \left. - (S \times \mathcal{E})(\gamma-\gamma' + \frac{2mc^2}{\hbar}) \right\} - (\gamma+\gamma') \left\{ S\gamma' (n \times \mathcal{E}, n') \right. \right. \\ \left. \left. + (n\gamma - n'\gamma')(S, n \times \mathcal{E}) \right\} - 2\gamma' (n' \mathcal{E}) [S \times (n\gamma - \gamma'n')] \right. \\ \left. + i \left\{ n\gamma' (n' \mathcal{E})(\gamma-\gamma') + \frac{(\gamma+\gamma')^2}{\gamma} mc \mathcal{E} - \frac{2\hbar\gamma'^2}{c} n' (n' \mathcal{E}) \right\} d \right]$$

$$= \frac{e^2}{\hbar} (2\pi\hbar)^3 \frac{1}{2mc\gamma\gamma' (\frac{2mc^2}{\hbar} + \gamma - \gamma')} \left[(\gamma-\gamma') \left\{ (n \times \mathcal{E})(S, \omega) - [S \times \mathcal{E}](\gamma-\gamma' + \frac{2mc^2}{\hbar}) \right\} \right. \\ \left. - (\gamma+\gamma') \left\{ S\gamma' (n \times \mathcal{E}, n') + \omega(S, n \times \mathcal{E}) \right\} - 2\gamma' (n' \mathcal{E}) [S \times \omega] \right. \\ \left. - i \left\{ n\gamma' (n' \mathcal{E})(\gamma-\gamma') + \frac{(\gamma+\gamma')^2}{\gamma} mc \mathcal{E} - \frac{2\hbar\gamma'^2}{c} n' (n' \mathcal{E}) \right\} d \right]$$

$$H' = - \frac{i\gamma'}{c} [n' \times A']$$

$$= - \frac{e^2}{\hbar} (2\pi\hbar)^3 \frac{i}{2mc^2\gamma (\gamma - \gamma' + \frac{2mc^2}{\hbar})} \left[(\gamma-\gamma') \left\{ [n' \times [n \times \mathcal{E}]](S, \omega) - [n' \times [S \times \mathcal{E}]] \right. \right. \\ \left. \left. \cdot (\gamma-\gamma' + \frac{2mc^2}{\hbar}) \right\} - (\gamma+\gamma') \left\{ \gamma' [n' \times S](n \times \mathcal{E}, n') + [n' \times \omega](S, n \times \mathcal{E}) \right\} \right. \\ \left. - 2\gamma' (n' \mathcal{E}) [n' \times [S \times \omega]] - i \left\{ \gamma' [n' \times n](n' \mathcal{E})(\gamma-\gamma') + \frac{(\gamma+\gamma')^2}{\gamma} mc^2 [n' \times \mathcal{E}] \right\} \right]$$

$-S(n' \times \omega) + \omega(n' \times S)$ $n' (n\gamma' - n\gamma)$

$$\propto \frac{1}{\gamma\gamma'} \left\{ (\gamma-\gamma') \left[(S\omega) \left((n'\epsilon)n - (nn')\epsilon \right) + K \left((Sn')\epsilon - (n'\epsilon)S \right) \right] - (\gamma+\gamma') \left[(n[\epsilon S]) \gamma^* [n'n] + \gamma' (n'[n\epsilon]) [n'S] \right] \right. \\ \left. + 2\gamma' (n'\epsilon) \left[(n'S)\omega + S(\gamma' - (nn')\gamma) \right] \right\}$$

$$\frac{1}{\gamma\gamma'} \left\{ (\gamma-\gamma') \left[(\tilde{S}, \omega) \left((n'\epsilon)n - (nn')\epsilon \right) + K \left((\tilde{S}n')\epsilon - (n'\epsilon)\tilde{S} \right) \right] - (\gamma+\gamma') \left[(n[\epsilon\tilde{S}]) \gamma^* [n'n] + \gamma' (n'[n\epsilon]) [n'\tilde{S}] \right] \right. \\ \left. + 2\gamma' (n'\epsilon) \left[(n'\tilde{S})\omega + \tilde{S}(\gamma' - (nn')\gamma) \right] \right\}$$

$$\gamma^2 \gamma'^2 \left[(\gamma-\gamma')^2 \left\{ \omega^2 \gamma \left((n'\epsilon)^2 + (nn')^2 \epsilon^2 \right) + K^2 \left(\epsilon^2 + 3(n'\epsilon)^2 - 2\epsilon^2 \right) - 2K \gamma \left((n'\omega)(nn')\epsilon^2 + (n\omega)(n'\epsilon)^2 - (\omega\epsilon)(n'\epsilon)(nn') \right) \right. \right. \right. \\ \left. \left. + (\gamma+\gamma')^2 \right\} \gamma'^2 (1 - (nn')^2) \left\{ \epsilon^2 + 2\gamma'^2 (n'[n\epsilon])^2 - 2\gamma'^2 (n'[n\epsilon])^2 (nn') \right\} \right. \\ \left. + 4\gamma'^2 (n'\epsilon)^2 \gamma \left\{ \omega^2 + 3\gamma' (\gamma' - (nn')\gamma)^2 + 2(n'\omega)(\gamma' - (nn')\gamma) \right\} \right. \\ \left. - (\gamma^2 \gamma'^2) \left\{ -2\gamma' (nn') (n'[n\epsilon]) (\omega[n\epsilon]) + 2\gamma' (n'[n\epsilon]) \left\{ (n'\epsilon)(\omega[nn']) - (nn')(\omega[\epsilon n']) \right\} \right. \right. \right. \\ \left. \left. + 2K\gamma' \left((\epsilon[n'n])^2 + (n'\epsilon)^2 \right) \right\} \right. \\ \left. + 2\gamma' (\gamma-\gamma') (n'\epsilon) \left\{ 2K \left((\omega\epsilon) + (n'\epsilon)(n'\omega) \right) \right\} \right. \\ \left. - 2\gamma' (\gamma+\gamma') (n'\epsilon) \left\{ 2\gamma' (n'[n\epsilon]) (\omega[n'n]) + 2\gamma' (n'\omega)(n'\epsilon) \right\} \right]$$

$$3(n'\omega)^2 - 2(n'\omega)^2$$

$$\omega^2 = (n\gamma - n'\gamma')^2$$

$$= \gamma^2 + \gamma'^2 - 2\gamma\gamma'(nn')$$

$$= (\gamma-\gamma')^2 + \frac{2m^2c^2(\gamma-\gamma')}{h} = (\gamma-\gamma') \left\{ \gamma - \gamma' + \frac{2m^2c^2}{h} \right\} = (\gamma-\gamma') K$$

$$mc^2 \left(\frac{\gamma' - \gamma}{h} \right) + \frac{h\gamma\gamma'}{mc^2} = \frac{h\gamma\gamma'}{mc^2} (nn')$$

$$\begin{aligned}
& \frac{d\bar{d}}{(\gamma\gamma')^2} \left\{ \gamma'^2 (n' \epsilon)^2 (\gamma - \gamma')^2 (1 - (nn')^2) + \cancel{\gamma'^2} \frac{(\gamma + \gamma')^4}{\gamma^2} \frac{m^2 c^4}{h^2} (\epsilon^2 - (n' \epsilon)^2) + 2 \frac{\cancel{\gamma'^2}}{\gamma^2} (\gamma + \gamma')^2 (\gamma - \gamma') (nn') (n' \epsilon)^2 \right\} \\
& = \frac{\cancel{\gamma}}{(\gamma\gamma')^2} \left\{ \epsilon^2 \frac{(\gamma + \gamma')^4 m^2 c^4}{h^2 \gamma^2} + (n' \epsilon)^2 \left\{ \gamma'^2 (\gamma - \gamma')^2 (1 - (nn')^2) - \frac{(\gamma + \gamma')^4 m^2 c^4}{h^2 \gamma^2} - \frac{2 \cancel{\gamma'^2}}{\gamma^2} (\gamma + \gamma')^2 (\gamma - \gamma') (nn') \frac{m^2 c^4}{h^2} \right\} \right\}
\end{aligned}$$

S-terms

$$\begin{aligned}
& \frac{\cancel{\gamma}}{(\gamma\gamma')^2} \left[(\gamma - \gamma')^2 \left\{ \cancel{K} (\gamma - \gamma') \left((n' \epsilon)^2 + \epsilon^2 \right) + K^2 (\epsilon^2 + (n' \epsilon)^2) - 2 \cancel{K} (\gamma - \gamma') (n' \epsilon)^2 \gamma \right\} \right. \\
& \quad + (\gamma + \gamma')^2 \left\{ \epsilon^2 \gamma^2 (1 - (nn')^2) + 2 \gamma'^2 (nn')^2 \right. \\
& \quad \quad \left. \left\{ \epsilon^2 (1 - (nn')^2) - (n' \epsilon)^2 \right\} - 2 \gamma \gamma' (nn') \left\{ \epsilon^2 (1 - (nn')^2) - (n' \epsilon)^2 \right\} \right\} \\
& \quad + 4 \gamma'^2 (n' \epsilon)^2 \left\{ \cancel{K} (\gamma - \gamma') + (n' \omega)^2 \right\} \\
& \quad - 2 K \gamma (\gamma^2 - \gamma'^2) \left\{ \epsilon^2 (1 - (nn')^2) - (n' \epsilon)^2 + (n' \epsilon)^2 \right\} \\
& \quad \left. + 4 K \gamma' (\gamma - \gamma') (n' \epsilon)^2 \left\{ (nn') \gamma - 2 \gamma' \right\} - 4 \gamma \gamma' (\gamma + \gamma') (n' \epsilon)^2 \left\{ (nn') \gamma - \gamma' \right\} \right]
\end{aligned}$$

$$\frac{e^2}{(vv')^2} \left[\frac{(v+v')^4}{\alpha^2} + (v-v')^2 \left\{ k(v-v') + k^2 - 2k(nn') \frac{(n'\omega)^2}{2} \right\} \right. \\ \left. + (v+v')^2 (1-(nn')^2) \left\{ v^2 + 2v'^2 - 2vv' \frac{(nn')^2}{2} \right\} - 2kv(v^2 - v'^2) \right] \\ + \frac{(n'e)^2}{(vv')^2} \left[-\frac{(v+v')^4}{\alpha^2} + \frac{v'^2}{2} (v-v')^2 (1-(nn')^2) - \frac{2v'}{v} (v+v')^2 (v-v') (nn') \frac{mc^2}{h} \right. \\ \left. + (v-v')^2 \left\{ k(v-v') + k^2 - 2kv \right\} \right. \\ \left. + 2(v+v')^2 \left\{ \frac{1}{2} vv' (nn') - \frac{1}{2} v'^2 \right\} + 4v'^2 (k(v-v') + (n'\omega)^2) \right. \\ \left. + 4kv'(v-v') \left\{ (nn')v - 2v' \right\} - 4vv'(v+v') ((nn')v - v') \right] \cdot$$

$$= \frac{e^2}{(vv')^2} \left[\frac{(v+v')^4}{\alpha^2} + k(v-v') \left\{ v^2 (1-(nn')^2) + v'^2 (1-(nn')^2) \right\} \right. \\ \left. + (v+v') \left\{ (v+v')v'^2 - k(v-v')^2 \right\} (1-(nn')^2) \right]$$

$$+ \frac{(n'e)^2}{(vv')^2} \left[-\frac{(v+v')^4}{\alpha^2} + (v-v')^2 \left\{ v'^2 (1-(nn')^2) - 2k \left(\frac{mc^2}{h} - v' \right) \right\} \right. \\ \left. + 2v'(v+v')^2 \left\{ (nn')(v-v') - v'(1-(nn')^2) \right\} \right. \\ \left. - 4vv'^2 ((nn')v - v') (1+(nn')) \right]$$

$$= \frac{e^2}{(vv')^2} \left[\frac{(v+v')^4}{\alpha^2} + (1-(nn'))v'^2 \left\{ 2(v+v')^2 + (v-v')^2 (1+(nn')) \right\} \right]$$

$$+ \frac{(n'e)^2}{(vv')^2} \left[-\frac{(v+v')^4}{\alpha^2} + (v-v')^2 \left\{ v'^2 (1-(nn')^2) - 2k \left(\frac{mc^2}{h} - v' \right) \right\} \right. \\ \left. + 2v'(v+v')^2 \left\{ (nn')(v-v') - v'(1-(nn')^2) \right\} \right. \\ \left. - 4vv'^2 ((nn')v - v') (1+(nn')) \right]$$

$$= \frac{e^2}{(vv')^2} \left[\frac{(v+v')^4}{\alpha^2} \right]$$

$$\frac{4}{\alpha^2} \left(\frac{v'}{v} \right)^2 \left(\frac{1}{1-(nn')} \right)$$

$$\frac{4}{\alpha^2} \left(\frac{v'}{v} \right)^2 \frac{1}{\left(\frac{v}{v'} - 1 \right)}$$

$$\frac{4v'^2}{\alpha^2}$$

$$\left(1 + \left(\frac{v}{v'} \right)^2 \right) \left(1 + \frac{v^2}{v'^2} \right)$$

$$- 2 \frac{v}{v'} \frac{1}{(nn')}$$

$$- \frac{8}{\alpha^2} \frac{v'}{v} \frac{1}{\left(\frac{v}{v'} - 1 \right)}$$

$$\frac{8}{\alpha^2} \left(1 + \frac{v^2}{v'^2} - \frac{2v}{v'} (nn') \right)$$

$$+ 2(nn') \left(\frac{v'}{v} - (nn') \right)$$

$$\frac{(n' \epsilon)^2}{(\gamma \gamma')^2} \left\{ - (1 - (n \omega)^2) \frac{\gamma^2 \gamma'^2 (\gamma + \gamma')^4}{(\gamma - \gamma')^2} + \gamma'^2 (\gamma - \gamma')^2 (1 - (n \omega)^2) - 2 \gamma'^2 (\gamma + \gamma')^2 (n \omega') (1 - (n \omega')) \right\}$$

$$\textcircled{X} + \frac{(\gamma - \gamma')^2 K (\gamma - \gamma' + K - 2\gamma)}{\Delta} - \frac{2(\gamma - \gamma')^2 \gamma (-\gamma' (n \omega') + \gamma) K}{\Delta} \quad \begin{matrix} \downarrow \\ (n \omega) \end{matrix}$$

$$+ 2(\gamma + \gamma')^2 \gamma' \left\{ \gamma (n \omega') - \gamma' \right\} \quad \begin{matrix} \downarrow \\ (n \omega) \end{matrix}$$

$$\textcircled{X} + 4 \frac{\gamma'^2 \{ K (\gamma - \gamma') + (n \omega')^2 \}}{\Delta} \quad \begin{matrix} \downarrow \\ (n \omega) \end{matrix}$$

$$\textcircled{\Delta} + 4 \frac{K \gamma' (\gamma - \gamma') \{ (n \omega) - \gamma' \}}{\Delta} - 4 \gamma \gamma' (\gamma + \gamma') (n \omega) \quad \begin{matrix} \downarrow \\ (n \omega) \end{matrix}$$

$$\begin{aligned} & (\gamma - \gamma') K \left\{ -2(\gamma - \gamma') (n \omega) + 4\gamma'^2 + 4\gamma' \{ (n \omega) - \gamma' \} \right\} \\ & - 2(\gamma - \gamma') (\gamma - \gamma' (n \omega')) + 4\gamma' (\gamma (n \omega') - \gamma') \\ & = -2\gamma^2 + 2\gamma \gamma' (n \omega') + 2\gamma \gamma' - 2\gamma'^2 (n \omega') \\ & \quad + 4\gamma \gamma' (n \omega') - 4\gamma'^2 = -2(\gamma^2 - \gamma \gamma' + 2\gamma'^2) \\ & 2(\gamma + \gamma') \left\{ (\gamma + \gamma') \gamma' \{ \gamma (n \omega') - \gamma' \} - 2(n \omega') (\gamma' - \gamma \gamma') \right\} \\ & \quad (n \omega) - 2\gamma \gamma' (n \omega) \\ & \quad \gamma'^2 (n \omega) - \gamma \gamma' (n \omega) \\ & = 2\gamma \gamma' (\gamma + \gamma') \left\{ \frac{\gamma'}{\gamma} ((n \omega') \gamma - \gamma') - ((n \omega') \gamma - \gamma') \right\} \end{aligned}$$

$$K(\gamma - \gamma') \left\{ (\gamma - \gamma') \right\}$$

$$-3^4 + 1 - 2(5) - 2(3)^2 + 4 \times 2$$

$$-81 + 1 - 10 - 18 + 8 = -100$$

$$-10 - 10$$

4

$$(\gamma - \gamma')^2 K(\gamma - \gamma' + K - 2\gamma) + 4\gamma'^2 K(\gamma - \gamma') + 4K\gamma'(\gamma - \gamma')\left\{ \frac{(n'\omega) - \gamma'}{2} \right\}$$

$$\frac{\gamma - \gamma' + \frac{c}{\omega}}{2}$$

8

$$= (\gamma - \gamma') K \left\{ (\gamma - \gamma') \left(\frac{2\omega^2}{h} - 2\gamma' \right) + 4\gamma'^2 + 4\gamma' \left(\frac{(n'\omega) - \gamma'}{2} \right) \right\}$$

$$= (\gamma - \gamma') K \left\{ (\gamma - \gamma') \left\{ \frac{2\gamma\gamma'}{\gamma - \gamma'} (1 - nu') - 2\gamma' \right\} + 4\gamma'^2 + 4\gamma' \left(\frac{(n'\omega) - \gamma'}{2} \right) \right\}$$

$$= (\gamma - \gamma') K \left\{ 2\gamma\gamma' (1 - nu') - 2\gamma'(\gamma - \gamma') + 4\gamma' \left\{ \frac{(n'\omega) - \gamma'}{2} \right\} \right\}$$

$$= (\gamma - \gamma') K \left\{ 2\gamma\gamma' - 2\gamma\gamma'(nu') - 2\gamma\gamma' + 2\gamma'^2 + 4\gamma\gamma'(nu') - 4\gamma'^2 \right\}$$

$$= 2(\gamma - \gamma') K(n'\omega)\gamma'$$

$$2(\gamma + \gamma')^2 \gamma' (n'\omega) + 4\gamma'^2 (n'\omega)^2 - 4\gamma\gamma'(\gamma + \gamma')(n'\omega)$$

$$= 2\gamma' (n'\omega) \left\{ (\gamma + \gamma')^2 + 2\gamma' (n'\omega) - 2\gamma(\gamma + \gamma') \right\}$$

$$= 2\gamma' (n'\omega) \left\{ \cancel{\gamma^2} + \cancel{2\gamma\gamma'} + \cancel{\gamma'^2} + 2\gamma\gamma'(nu') - \cancel{\gamma^2} - \cancel{2\gamma\gamma'} - \cancel{\gamma'^2} \right\}$$

$$= -2\gamma' (n'\omega) K(\gamma - \gamma')$$

$$\therefore (\gamma - \gamma')^2 K(\gamma - \gamma' + K - 2\gamma) + 4\gamma'^2 K(\gamma - \gamma') + 2(\gamma + \gamma')^2 \gamma' \left\{ \frac{(n'\omega) - \gamma'}{2} \right\}$$

$$+ 4\gamma'^2 \left\{ K(\gamma - \gamma') + (n'\omega)^2 \right\} + 4K\gamma'(\gamma - \gamma') \left\{ \frac{(n'\omega) - \gamma'}{2} \right\} - 4\gamma\gamma'(\gamma + \gamma')(n'\omega) = 0$$

$$- (1 - (nu'))^2 \frac{\gamma'^2 (\gamma + \gamma')^4}{(\gamma - \gamma')^2} + \gamma'^2 (\gamma - \gamma')^2 (1 - (nu')^2) - 2\gamma'(\gamma + \gamma')^2 (nu')(1 - (nu'))$$

$$= (1 - (nu')) \gamma'^2 \left\{ - (1 - (nu')) \frac{(\gamma + \gamma')^4}{(\gamma - \gamma')^2} + (\gamma - \gamma')^2 - 2 \frac{(1 + nu')}{(\gamma + \gamma')^2 (nu')} \right\}$$

$$= (1 - (nu')) \gamma'^2 \left\{ (nu') \left\{ \frac{(\gamma + \gamma')^4}{(\gamma - \gamma')^2} - 2(\gamma + \gamma')^2 \right\} - \frac{(\gamma + \gamma')^4}{(\gamma - \gamma')^2} + (\gamma - \gamma')^2 \right\}$$

$$= \frac{(1 - (nu')) \gamma'^2}{(\gamma - \gamma')^2} \left\{ (nu') \left\{ (\gamma + \gamma')^4 - 2(\gamma^2 - \gamma'^2)^2 \right\} + \left\{ (\gamma^2 - \gamma'^2)^2 - (\gamma + \gamma')^4 \right\} \left\{ \frac{(\gamma - \gamma')^2 + (\gamma + \gamma')^2}{2(\gamma^2 + \gamma'^2)} \right\} \right\}$$

$$\left\{ \frac{(\gamma + \gamma')^4 - (\gamma - \gamma')^4}{(\gamma + \gamma')^2 - (\gamma - \gamma')^2} \right\}^2 = \frac{(4\gamma\gamma')^2}{(4\gamma\gamma')^2} = \frac{1}{6} \gamma^2 \gamma'^2$$

$$- 8\gamma\gamma'(\gamma^2 + \gamma'^2)$$

$$= 8 \frac{(1-uu') \gamma \gamma'^3}{(\gamma-\gamma')^2} \left\{ 2\gamma\gamma'(uu') \ominus \gamma^2 + \gamma'^2 \right\} = \frac{-8(1-uu') \gamma \gamma'^3}{(\gamma-\gamma')} K.$$

$$\frac{mc^2}{h\nu} = \frac{\gamma'}{\gamma-\gamma'} (1-uu') = \frac{1}{\alpha}. \quad (\gamma+\gamma')^2 - (\gamma-\gamma')^2 = 4\gamma\gamma' \quad \left\{ \frac{(\gamma+\gamma')^2 - (\gamma-\gamma')^2}{4\gamma\gamma'} (uu') \right\}$$

$$\frac{1}{\alpha^2} = \frac{\gamma'^2}{(\gamma-\gamma')^2 (1-uu')^2}$$

$$= \frac{8\gamma\gamma'}{\alpha^2 (1-uu')} \left\{ 2\gamma\gamma'(uu') \ominus \gamma^2 + \gamma'^2 \right\}$$

$$= -\frac{8\gamma\gamma'^2}{\alpha} K.$$

$$E^2 \text{ term} = \frac{4(1+(\frac{\gamma}{\gamma'})^2)}{\alpha^2 (1-uu')} \left(1 + \frac{\gamma^2}{\gamma'^2} - 2\frac{\gamma}{\gamma'} (uu') \right) \frac{\gamma'^2}{\gamma}$$

$$= \frac{4(1+(\frac{\gamma}{\gamma'})^2)}{\alpha^2 (1-uu')} K (\gamma-\gamma') \frac{\gamma'^2}{\gamma} \quad \frac{\gamma-\gamma'}{1-uu'} = \alpha\gamma'$$

$$= \frac{4(1+(\frac{\gamma}{\gamma'})^2)}{\alpha} K \frac{\gamma'^3}{\gamma} \quad \frac{\gamma'}{\gamma} \times \frac{\gamma'}{\gamma' \pm \gamma}$$

$$\text{Total} = \frac{1}{\gamma\gamma'} \left(E^2 \text{ term} + \frac{1}{2}(1-uu')^2 (u' E)^2 \text{ term} \right)$$

$$= \frac{1}{\gamma\gamma'} \left(\frac{4(1+(\frac{\gamma}{\gamma'})^2)}{\alpha} K \frac{\gamma'^3}{\gamma} \ominus \frac{4\gamma\gamma'^2}{\alpha} K (1-uu')^2 \right)$$

$$= \frac{4K}{\alpha} \frac{\gamma'^2}{(\gamma\gamma')^2} \left\{ \frac{\gamma(1+uu')(1-uu')}{(1+(\frac{\gamma}{\gamma'})^2) \gamma'} \ominus \gamma(1-uu')^2 \right\}$$

$$= \frac{4K}{\alpha} \frac{1}{\gamma'\gamma^3} \left\{ \frac{\gamma}{\gamma'} + \frac{\gamma'}{\gamma} \ominus (1-uu')^2 \right\}$$

Total

$$\frac{4}{\alpha} \left(1 + \frac{v^2}{v'^2}\right) K \frac{v'}{v^2} - (1 - (nn')^2) \frac{4}{\alpha v} K$$

$$= \frac{4K}{\alpha v} \left\{ \frac{v'}{v} \left(1 + \frac{v^2}{v'^2}\right) - (1 - (nn')^2) \right\}$$

$$= \frac{4K}{\alpha v} \left\{ \frac{1}{vv'} (v^2 + v'^2) - (1 - (nn')^2) \right\}$$

$$= \frac{4K}{\alpha v} \left\{ \frac{v}{v'} + \frac{v'}{v} - (1 - (nn')^2) \right\}$$

factor $\frac{e^4}{4\pi\epsilon_0 v^2} \left(\frac{v'}{v}\right)^3 \frac{\alpha v}{K}$

$$1 - (nn')^2 = \frac{\beta}{\alpha} \left(2 - \frac{\beta}{\alpha}\right)$$

$$\frac{v'}{v} = \frac{1}{1+\beta} \quad \frac{v}{v'} = 1+\beta$$

$$\frac{2+2\beta+\beta^2}{(1+\beta)^2} = \frac{\alpha^2}{\alpha^2}$$

$$K = \frac{v(2+2\beta+\alpha\beta)}{\alpha(1+\beta)}$$

$$\frac{v}{v'} = 1+\beta \quad \frac{v'}{v} = \frac{1}{1+\beta}$$

$$= \frac{4(2+2\beta+\alpha\beta)}{\alpha^2(1+\beta)} \left\{ 1+\beta + \frac{1}{1+\beta} - \frac{\beta}{\alpha} \left(\frac{2\alpha-\beta}{\alpha}\right) \right\}$$

$$= \frac{4(2+2\beta+\alpha\beta)}{\alpha^2(1+\beta)} \left\{ \frac{2\alpha^2+2\alpha\beta+\beta^2}{\alpha^2(1+\beta)} - \frac{\beta(2\alpha-\beta)}{\alpha^2} \right\}$$

$$\times \frac{\alpha^2(1+\beta)}{8(2+2\beta+\alpha\beta)} = \frac{2\alpha-\beta}{2\alpha-\beta+2\alpha\beta}$$

$$= \frac{1}{2} \left\{ 1 + \frac{\beta}{1+\beta} \cdot \beta + \frac{1}{1+\beta} - \frac{\beta}{\alpha} \left(\frac{2\alpha-\beta}{\alpha}\right) \right\}$$

$$\frac{\{1+\alpha(1-nn')\}}{(1+(nn')^2)}$$

$$\frac{\alpha^2\beta(1+\beta) + \alpha^2 - \beta(2\alpha-\beta)(1+\beta)}{\alpha^2(1+\beta)} = \frac{\alpha^2\beta + \alpha^2\beta^2 + \alpha^2 - 2\alpha\beta + \beta^3 - 2\alpha\beta^2 + \beta^3}{\alpha^2(1+\beta)}$$

$$1 + (nn')^2 = 2 - \frac{2\beta}{\alpha} + \frac{\beta^2}{\alpha^2}$$

$$\beta^2 \frac{(\alpha^2 - 2\alpha\beta + \beta^2) + \alpha^2\beta^2}{\alpha^2(1+\beta)}$$

(100 裏)

$$\frac{1}{\lambda} + (1000 - 1) = \frac{1}{\lambda'} + 2 \left(\frac{1}{\lambda} + 1 \right)$$

$$\left(\frac{1}{\lambda} + 1 \right) = \left(\frac{1}{\lambda'} + 1 \right) + \frac{1}{\lambda} + 1$$

$$\left(\frac{1}{\lambda} + 1 \right) - \left(\frac{1}{\lambda'} + 1 \right) = \frac{1}{\lambda} + 1$$

$$\left(\frac{1}{\lambda} + 1 \right) - \frac{1}{\lambda'} = \frac{1}{\lambda} + 1$$

$$\left(\frac{1}{\lambda} - 1 \right) \lambda = 1000 - 1$$

$$\frac{1}{\lambda} - 1 = \frac{1}{\lambda'} - 1$$

$$1000(1 + \frac{1}{\lambda}) = 1$$

$$1 + \frac{1}{\lambda} = \frac{1}{\lambda'} + 1$$

$$\frac{1}{\lambda} - \frac{1}{\lambda'} = 0$$

$$\frac{1}{\lambda} - \frac{1}{\lambda'} = \frac{1}{\lambda} + 1$$

$$\lambda v = c$$

$$v = \frac{c}{\lambda}$$

$$\left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) = \frac{1}{\lambda} + 1$$

$$\frac{h\nu}{mc\lambda} = \frac{h}{mc\lambda} = \frac{6.54}{0.913 \times 10^{-31} \times 2.2 \times 10^{11}} = \frac{6.54}{2.7 \times 10^{-22}}$$

$$\frac{1}{\lambda} - \frac{1}{\lambda'} = 1000 + 1$$

$$\frac{1}{\lambda} - \frac{1}{\lambda'} = 1001$$

$$\left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) \lambda = 1001$$

$$1 - \frac{\lambda}{\lambda'} = 1001\lambda$$

$$1 - \frac{\lambda}{\lambda'} = 1001\lambda$$

$$(1 + 1)\lambda$$

$$(1000 - 1) + 1$$

$$I = \frac{(\cancel{4\pi})^6 e^4 v'^2}{4 m^2 c^4 \cancel{K^2} \lambda^2} \frac{1}{2(\cancel{2\pi})^6} \left(1 + \frac{mc^2}{E'}\right) \frac{E'}{mc^2} \frac{v'}{v} \left[\cancel{E'^2} \left[\right] + \frac{1}{2}(1 - (uu')^2) (u'_e)^2 \right]$$

$$= \frac{e^4}{m^2 c^4 \lambda^2} \frac{v'^3}{v^3} \frac{1}{8} \left(\frac{E'}{mc^2} + 1 \right) \frac{1}{K} \times \cancel{K}$$

$$\frac{1}{8} \frac{\cancel{4} h \phi}{mc^2} \frac{1}{K} \propto \frac{1}{8} K$$

$$= \frac{e^4}{m^2 c^4 \lambda^2} \frac{v'^3}{v^3} \frac{1}{8} \frac{\alpha^2 (1 + \beta)}{(2 + 2\beta + \alpha^2)}$$

$$E' = h\nu - h\nu' + mc^2$$

$$= h(\nu - \nu') + mc^2$$

$$\frac{E'}{h} = (\nu - \nu') + \frac{mc^2}{h} = K - \frac{mc^2}{h}$$

$$E' + mc^2 = h(\nu - \nu') + 2mc^2$$

$$= K h$$

$$\text{Coeff} = \frac{e^4}{m^2 c^4 \lambda^2} \left(\frac{v'}{v}\right)^3 \frac{1}{8} \frac{\alpha v}{K}$$

unpol. light

$$I = 2I_D \cdot \frac{1}{8} \frac{\alpha v}{K} \times \frac{\cancel{K}}{\cancel{2v}} \left\{ \frac{\nu}{\nu'} + \frac{\nu'}{\nu} - (1 - (uu')^2) \right\}$$

$$= I_D \left\{ \frac{\nu}{\nu'} + \frac{\nu'}{\nu} - (1 - (uu')^2) \right\} \frac{1}{1 + (uu')^2}$$

$$1 + (uu')^2 = 2$$

$$u'^2 = \frac{1 - \cos^2 \theta}{1 - (uu')^2}$$

$$= I_D \left\{ 1 + \frac{\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - 2}{1 + (uu')^2} \right\}$$

$$\frac{\nu'}{\nu} = \frac{1}{1 + \alpha(1 - (uu'))}$$

$$\frac{\nu}{\nu'} = 1 + \alpha(1 - (uu'))$$

$$= I_D \left\{ 1 + \frac{\alpha^2 (1 - uu')^2}{(1 + (uu')^2) (1 + \alpha(1 - uu'))} \right\}$$

$$I = \frac{e^4}{m^2 c^4 \lambda^2} \frac{v'^3}{v^3} \frac{1}{8} \frac{\alpha v}{K} \left[E'^2 \left[\right] + \frac{1}{2}(1 - (uu')^2) (u'_e)^2 \right] = \frac{1}{\dots} \left\{ \alpha^2 (1 - uu')^2 \dots \right\}$$

$$\therefore I = I_D \left\{ 1 + \frac{\alpha^2 (1 - uu')^2}{(1 + (uu')^2) (1 + \alpha(1 - uu'))} \right\}$$

③-1

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1.

$$[\beta_0 + (\alpha, \beta + \frac{e}{c} A) + \beta_3 mc] \psi = 0$$

$$\beta_0 = i\hbar \frac{\partial}{\partial t} \quad \beta_r = -i\hbar \frac{\partial}{\partial x_r}$$

$$\varphi [-\beta_0 + (\alpha, -\beta + \frac{e}{c} A) + \beta_3 mc] = 0$$

$$\beta_0 = \frac{E}{c}$$

$$[\beta_0 + (\alpha \beta) + \beta_3 mc] \psi = -\frac{e}{c} (\alpha A) \psi$$

$$\varphi [-\beta_0 - (\alpha \beta) + \beta_3 mc] = -\frac{e}{c} \varphi (\alpha A)$$

$$[-\beta_0 + (\alpha \beta) + \beta_3 mc] [\beta_0 + (\alpha \beta) + \beta_3 mc] \psi = -\frac{e}{c} [-\beta_0 + (\alpha \beta) + \beta_3 mc] (\alpha A) \psi$$

$$[-\beta_0^2 + \beta^2 + m^2 c^2] \psi = -\frac{e}{c} [-\beta_0 + (\alpha \beta) + \beta_3 mc] (\alpha A) \psi$$

$$\varphi [\beta_0 - (\alpha \beta) + \beta_3 mc] [-\beta_0 - (\alpha \beta) + \beta_3 mc] = -\frac{e}{c} \varphi [\beta_0 - (\alpha \beta) + \beta_3 mc] (\alpha A)$$

$$\varphi [-\beta_0^2 + \beta^2 + m^2 c^2] = -\frac{e}{c} \varphi [\beta_0 - (\alpha \beta) + \beta_3 mc] (\alpha A)$$

$$\left[\frac{\hbar^2}{c^2} \frac{\partial^2}{\partial t^2} - \hbar^2 \sum \frac{\partial^2}{\partial x_r^2} + m^2 c^2 \right] \psi = \frac{e}{c} \left[i\hbar \frac{\partial}{\partial t} + i\hbar (\alpha \text{grad}) + \beta_3 mc \right] (\alpha A) \psi$$

$$\psi_0 = u e^{-\frac{i}{\hbar} [Et - (mcr)]}$$

$$\varphi_0 = v e^{\frac{i}{\hbar} [Et - (mcr)]}$$

$$\psi = \psi_0 + \alpha f$$

$$\varphi = \varphi_0 + \alpha g$$

$$\alpha A = \alpha a e^{i\nu(t - \frac{nr}{c})} + \alpha \tilde{a} e^{-i\nu(t - \frac{nr}{c})}$$

$$\therefore \left[\frac{\hbar^2}{c^2} \frac{\partial^2}{\partial t^2} - \hbar^2 (\text{grad})^2 + m^2 c^2 \right] f = \frac{e}{c} \left[i\hbar \frac{\partial}{\partial t} + i\hbar (\alpha \text{grad}) + \beta_3 mc \right] (\alpha A) \psi_0$$

$$(\alpha A) \psi_0 = (\alpha a)_u e^{-\frac{i}{\hbar} [(E - \hbar\nu)t - (m - \frac{\hbar\nu}{c} n) \cdot r]} + (\alpha \tilde{a})_{\tilde{u}} e^{-\frac{i}{\hbar} [(E + \hbar\nu)t - (m + \frac{\hbar\nu}{c} n) \cdot r]}$$

$$\text{Setze: } f = F_1 e^{-\frac{i}{\hbar} [Et - m_1 r]} + F_2 e^{-\frac{i}{\hbar} [E_2 t - m_2 r]}$$

$$\therefore \left(-\frac{E_1^2}{c^2} + m_1^2 + m^2 c^2 \right) F_1 = \frac{e}{c} \left[\frac{E_1}{c} - (\alpha m_1) - \beta_3 mc \right] (\alpha a)_u$$

$$\left(-\frac{E_2^2}{c^2} + m_2^2 + m^2 c^2\right) F_2 = \frac{e}{c} \left[\frac{E_2}{c} - (\alpha m_2) - \beta mc \right] (\alpha \tilde{a}) u$$

$$\begin{aligned} -\frac{E_2^2}{c^2} + m_2^2 + m^2 c^2 &= -\frac{(E_0 - h\nu)^2}{c^2} + \left(m_0 - \frac{h\nu}{c} n\right)^2 + m^2 c^2 \\ &= -\left(\frac{E_0^2}{c^2} - \frac{2E_0 h\nu}{c^2} + \left(\frac{h\nu}{c}\right)^2\right) + \left(m_0^2 + \left(\frac{h\nu}{c}\right)^2 - 2\frac{h\nu}{c}(nm_0)\right) + m^2 c^2 \\ &= \frac{2h\nu}{c} \left(\frac{E_0}{c} - (nm_0)\right) \end{aligned}$$

$$-\frac{E_2^2}{c^2} + m_2^2 + m^2 c^2 = -\frac{2h\nu}{c} \left(\frac{E_0}{c} - (nm_0)\right)$$

$$\therefore F_1 = \frac{e}{2h\nu \left(\frac{E_0}{c} - (nm_0)\right)} \left[\frac{E_1}{c} - (\alpha m_1) - \beta mc \right] (\alpha a) u$$

$$F_2 = \frac{-e}{2h\nu \left(\frac{E_0}{c} - (nm_0)\right)} \left[\frac{E_2}{c} - (\alpha m_2) - \beta mc \right] (\alpha \tilde{a}) u$$

$$\begin{aligned} \mathcal{G} \left[\frac{\hbar^2 \partial^2}{c^2 \partial t^2} - \hbar^2 (\text{grad})^2 + m^2 c^2 \right] &= -\frac{e}{c} \varphi_0 \left[\overset{(\alpha A)}{\cancel{p_0}} - (\alpha \cancel{p}) + \beta mc \right] \\ &\quad - \frac{e}{c} \varphi_0 \left[i\hbar \frac{\partial}{c \partial t} + i\hbar (\alpha \text{grad}) + \beta mc \right] (\alpha \tilde{A}) \\ \varphi_0(\alpha A) &= V(\alpha A) e^{\frac{i}{\hbar} \left[\overset{E_2}{(E+h\nu)} t - \left(\overset{m_2}{m + \frac{h\nu}{c} n} \right) r \right]} + V(\alpha \tilde{a}) e^{\frac{i}{\hbar} \left[\overset{E_1}{(E-h\nu)} t - \left(\overset{m_1}{m - \frac{h\nu}{c} n} \right) r \right]} \end{aligned}$$

$$\text{Setze } \mathcal{G} = G_1 e^{\frac{i}{\hbar} \left[\overset{E_1}{(E-h\nu)} t - \left(\overset{m_1}{m - \frac{h\nu}{c} n} \right) r \right]} + G_2 e^{\frac{i}{\hbar} [E_2 t - (m_2 r)]}$$

$$G_1 \left(-\frac{E_1^2}{c^2} + m_1^2 + m^2 c^2 \right) = -\frac{e}{c} V \left[-\frac{E_1}{c} + (\alpha m_1) + \beta mc \right] (\alpha \tilde{a})$$

$$G_2 \left(-\frac{E_2^2}{c^2} + m_2^2 + m^2 c^2 \right) = -\frac{e}{c} V \left[-\frac{E_2}{c} + (\alpha m_2) + \beta mc \right] (\alpha \tilde{a})$$

$$\therefore G_1 = \frac{-e}{2h\nu \left(\frac{E_0}{c} - (nm_0)\right)} V \left[-\frac{E_1}{c} + (\alpha m_1) + \beta mc \right] (\alpha \tilde{a})$$

$$G_2 = \frac{e}{2h\nu \left(\frac{E_0}{c} - (nm_0)\right)} V \left[-\frac{E_2}{c} + (\alpha m_2) + \beta mc \right] (\alpha \tilde{a})$$

$$J = cs = ice \varphi' \alpha \psi = ice (\varphi'_0 \alpha \psi_0 + \alpha g' \alpha \psi_0 + \alpha \varphi'_0 \alpha f)$$

3

$$\psi_0 = u e^{-\frac{i}{\hbar}[Et - (m'r)]} \quad \varphi'_0 = v' e^{-\frac{i}{\hbar}[E't - (m'e'r)]}$$

$$f = F_1 e^{-\frac{i}{\hbar}[Et - (m'r)]} + F_2 e^{-\frac{i}{\hbar}[E_2 t - (m_2'r)]} \quad g' = G_1' e^{-\frac{i}{\hbar}[E_1' t - (m_1'e'r)]} + G_2' e^{-\frac{i}{\hbar}[E_2' t - (m_2'e'r)]}$$

$$g' \alpha \psi_0 = G_1' \alpha u e^{-\frac{i}{\hbar}[(E-E_1')t - (m-m_1')r]} + G_2' \alpha u e^{-\frac{i}{\hbar}[(E_2-E)t - (m_2'-m)r]}$$

$$\varphi'_0 \alpha f = v' \alpha F_1 e^{-\frac{i}{\hbar}[(E-E_1')t - (m-m_1')r]} + v' \alpha F_2 e^{-\frac{i}{\hbar}[(E_2-E)t - (m_2'-m)r]}$$

$$J_{ice} = \int d\mathcal{M}' \left\{ (G_1' \alpha u + v' \alpha F_2) e^{-\frac{i}{\hbar}[(E+kv-E')t - (m+\frac{kv}{c}n-m')r]} \right. \\ \left. + (G_2' \alpha u + v' \alpha F_1) e^{-\frac{i}{\hbar}[(E'+kv-E)t - (m'+\frac{kv}{c}n-m)r]} \right\}$$

$$\therefore J_{ice} = \int d\mathcal{M}' \left\{ (G_2' \alpha u' + v' \alpha F_1') e^{-\frac{i}{\hbar}[(E+kv-E')t - (m+\frac{kv}{c}n-m')r]} \right. \\ \left. + (G_1' \alpha u + v' \alpha F_2) e^{-\frac{i}{\hbar}[(E'+kv-E)t - (m'+\frac{kv}{c}n-m)r]} \right\}$$

$$Q = \frac{ie}{R} \int dv \int d\mathcal{M}' \left\{ \begin{aligned} & () e^{-\frac{i}{\hbar}[(E+kv-E')(t-\frac{R}{c} + \frac{n'r}{c}) - (m+\frac{kv}{c}n-m')r]} \\ & () e^{-\frac{i}{\hbar}[(E'+kv-E)(t-\frac{R}{c} + \frac{n'r}{c}) - (m'+\frac{kv}{c}n-m)r]} \end{aligned} \right\}$$

$$= \frac{ie}{R} (2\pi\hbar)^3 \left\{ (G_2' \alpha u' + v' \alpha F_1') e^{i v' (t - \frac{R}{c})} + (G_1' \alpha u + v' \alpha F_2) e^{-i v' (t - \frac{R}{c})} \right\}$$

$$F_1' = \frac{e}{2\hbar v' (\frac{E_1'}{c} - (n m_1'))} \left[\frac{E_1'}{c} - (\alpha m_1') - \beta_1 mc \right] (\alpha a) u' \quad F_2' = \frac{-e}{2m c \hbar v'} \left[\frac{E_2}{c} - \frac{\hbar v}{c} (n \alpha) \right. \\ \left. - \beta_2 mc \right] (\alpha \tilde{a}) u'$$

$$= \frac{e}{2m c \hbar v'} \left[\frac{E_1'}{c} + \frac{\hbar v'}{c} (n' \alpha) - \beta_1 mc \right] (\alpha a) u' \quad G_2' = \frac{-e}{2m c \hbar v'} v' \left[+ \frac{E_2}{c} - \frac{\hbar v}{c} (n \alpha) \right. \\ \left. - \beta_2 mc \right] (\alpha a)$$

$$G_1' = \frac{+e}{2m c \hbar v'} v' \left[+ \frac{E_1'}{c} + \frac{\hbar v'}{c} (n' \alpha) - \beta_1 mc \right] (\alpha \tilde{a})$$

$$\begin{aligned}
 G_2 \alpha u' &= -\frac{e}{2mch\nu} \nu \left[\frac{E_2}{c} - \frac{h\nu}{c} (n\alpha) - \beta mc \right] (\alpha a) \alpha u' \\
 &= -\frac{e}{2mch\nu} \nu \left[\frac{E_2}{c} (\alpha a) \alpha - \frac{h\nu}{c} (n\alpha) (\alpha a) \alpha - \beta mc (\alpha a) \alpha \right] u'
 \end{aligned}$$

$$\begin{aligned}
 \nu \alpha F_1' &= \frac{e}{2mch\nu'} \nu \alpha \left[\frac{E_1'}{c} + \frac{h\nu'}{c} (n'\alpha) - \beta mc \right] (\alpha a) u' \\
 &= \frac{e}{2mch\nu'} \nu \left[\frac{E_1'}{c} \alpha (\alpha a) + \frac{h\nu'}{c} \alpha (n'\alpha) (\alpha a) - \alpha \beta mc (\alpha a) \right] u'
 \end{aligned}$$

$$\begin{aligned}
 &\frac{\nu E_1'}{c} \alpha_1 (\alpha a) - \frac{\nu E_2'}{c} (\alpha a) \alpha_1 \\
 &= \frac{\nu E_1'}{c} \alpha_1 (\alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3) - \frac{\nu E_2'}{c} (\alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3) \alpha_1 \\
 &= \frac{1}{c} \left\{ \nu E_1' a_1 + i \nu E_1' (a_2 \sigma_3 - a_3 \sigma_2) - \nu E_2' a_1 - i E_2' \nu (-a_2 \sigma_3 + a_3 \sigma_2) \right\} \\
 &= \frac{1}{c} \left\{ i (\nu E_1' + E_2') [a \sigma], + a_1 (\nu E_1' - E_2') \right\} \\
 &= \frac{1}{c} \left\{ i \frac{mc^2(\nu + \nu')}{2} [a \sigma], + i h \nu \nu' [a \sigma], - h(\nu + \nu') a_1 \right. \\
 &\quad \left. + (mc^2(\nu - \nu') + 2h\nu\nu') a_1 \right\}
 \end{aligned}$$

$$\frac{h\nu\nu'}{c} (n\alpha) (\alpha a) \alpha_1 + \frac{h\nu\nu'}{c} \alpha_1 (n'\alpha) (\alpha a)$$

$$\begin{aligned}
 &= \frac{h\nu\nu'}{c} \left\{ (n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3) (a_1 \alpha_1 + a_2 \alpha_2 + a_3 \alpha_3) \alpha_1 \right. \\
 &\quad \left. + \alpha_1 (n_1' \alpha_1 + n_2' \alpha_2 + n_3' \alpha_3) (a_1 \alpha_1 + a_2 \alpha_2 + a_3 \alpha_3) \right\}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{h\nu\nu'}{c} \left\{ (n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3) (a_1 + i a_2 \sigma_3 + i a_3 \sigma_2) \right. \\
 &\quad \left. + (n_1' \alpha_1 - n_2' \alpha_2 - n_3' \alpha_3) (a_1 + i a_2 \sigma_3 - i a_3 \sigma_2) \right\}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{h\nu\nu'}{c} \left\{ \underbrace{(n_1 + n_1') \alpha_1 a_1}_{n_2 a_1 - n_1 a_2} + \underbrace{(n_2 - n_2') \alpha_2 a_1}_{n_3 a_1 - n_1 a_3} + (n_3 - n_3') \alpha_3 a_1 \right. \\
 &\quad \left. - (n_1 - n_1') \alpha_1 a_2 + (n_2 + n_2') \alpha_2 a_1 - i (n_3 + n_3') \alpha_3 a_2 \right. \\
 &\quad \left. - (n_1 - n_1') \alpha_1 a_3 + i (n_2 + n_2') \alpha_2 a_3 + (n_3 + n_3') \alpha_3 a_3 \right\}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\hbar \nu \nu'}{c} \left\{ \alpha_1 (n'a_2) - [na]_3 \alpha_2 + [n'a]_3 \alpha_2 + [na]_2 \alpha_3 - [n'a]_2 \alpha_3 \right. \\
 &\quad \left. i\beta_1 [na] + i\beta_1 [n'a] \right\} \\
 &= \frac{\hbar \nu \nu'}{c} \left\{ \alpha_1 (n'a) + [na] \alpha_1 - [n'a] \alpha_1 + i\beta_1 [(n+n')a] \right\} \\
 &= \frac{\hbar \nu \nu'}{c} \left\{ \alpha_1 (n'a) + [na] \alpha_1 - [n'a] \alpha_1 + i\beta_1 [(n+n')a] \right. \\
 &\quad \left. + [\alpha[na]] + [\alpha[n'a]] + [\alpha[n'a]] - i[na] + i[n'a] \right\}
 \end{aligned}$$

$$\begin{aligned}
 &mc \nu' \beta (\alpha a) - mc \nu \beta (\alpha a) \\
 &= mc \left\{ \nu' \beta (\alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3) \alpha_1 - \nu \beta (\alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3) \right\} \\
 &= mc \left\{ \nu' \beta_3 a_1 + \nu \beta_3 a_1 - i \nu' \beta_3 \alpha_3 a_2 + i \nu' \beta_3 \alpha_2 a_3 + i \nu \beta_3 \alpha_3 a_2 - i \nu \beta_3 \alpha_2 a_3 \right\} \\
 &= mc \left\{ a_1 (\nu + \nu') - i \nu' [a \alpha] + i \nu [a \alpha] \right\} \\
 &= mc \beta_3 \left\{ a_1 (\nu + \nu') + i [a \alpha], (\nu - \nu') \right\}
 \end{aligned}$$

$$\begin{aligned}
 G_2 &\propto u' + v \propto F_1' \\
 &= \frac{e}{2mch \nu \nu'} \left[i mc \beta_3 [a \alpha] (\nu + \nu') + mc \beta_3 a (\nu - \nu') \right. \\
 &\quad \left. + mc a (\nu - \nu') + i mc \beta_3 [a \alpha] (\nu + \nu') - \frac{2 \hbar \nu \nu'}{c} a \right. \\
 &\quad \left. + \frac{\hbar \nu \nu'}{c} \beta_1 \left\{ \alpha (n'a) + [na] \alpha - [n'a] \alpha + i [na] + i [n'a] \right\} \right] i \\
 &\quad \left[\alpha [na] + \alpha [n'a] \right]
 \end{aligned}$$

$$\begin{aligned}
 \omega_{\beta_3} u' &= -v u' \\
 2 [nxa] (na) \beta_1 \frac{\hbar \nu \nu'}{c} &= 2 a (na) \beta_1 \frac{\hbar \nu \nu'}{c} + 2 i mc \nu [a \alpha] + 2 i mc \nu \beta_3 [a \alpha] \\
 \therefore [nxa] \beta_1 \frac{\hbar \nu \nu'}{c} &= i mc^2 \alpha (1 + \beta_3)
 \end{aligned}$$

$$\frac{\hbar \nu \nu'}{c} \left\{ (a_1 \alpha_1 + a_2 \alpha_2 + a_3 \alpha_3) (n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3) \alpha_1 \right. \\ \left. + \alpha_1 (n_1' \alpha_1 + n_2' \alpha_2 + n_3' \alpha_3) (\alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3) \right\}$$

$$= \frac{\hbar \nu \nu'}{c} \left\{ (a_1 \alpha_1 + a_2 \alpha_2 + a_3 \alpha_3) (n_1 - i n_2 \alpha_3 + i n_3 \alpha_2) \right. \\ \left. + (n_1' + i n_2' \alpha_3 - i n_3' \alpha_2) (\alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3) \right\}$$

$$= \frac{\hbar \nu \nu'}{c} \left\{ (a \alpha) (n_1 + n_1') - i n_2 \left\{ -i \alpha_2 a_1 + i \alpha_1 a_2 + \rho_1 a_3 \right\} \right. \\ \left. + i n_3 \left\{ i \alpha_3 a_1 + \rho_1 a_2 - i \alpha_1 a_3 \right\} \right. \\ \left. + i n_2' \left\{ i \alpha_2 a_1 - i \alpha_1 a_2 + \rho_1 a_3 \right\} \right. \\ \left. - i n_3' \left\{ -i \alpha_3 a_1 + \rho_1 a_2 + i \alpha_1 a_3 \right\} \right\}$$

$$= \frac{\hbar \nu \nu'}{c} \left\{ (a \alpha) (n_1 + n_1') + n_2 [\alpha a]_3 - n_3 [\alpha a]_2 - i \rho_1 [n a] \right. \\ \left. + n_2' [\alpha a]_3 - n_3 [\alpha a]_2 + i \rho_1 [n' a] \right\}$$

$$= \frac{\hbar \nu \nu'}{c} \left\{ (a \alpha) (n_1 + n_1') + [n \times [\alpha a]] - i \rho_1 [n a] \right. \\ \left. + [n' \times [\alpha a]] + i \rho_1 [n' a] \right\}$$

$$= \frac{\hbar \nu \nu'}{c} \rho_1 \left\{ (a \alpha) (n + n') + [n [\alpha a]] + [n' [\alpha a]] - i [n a] + i [n' a] \right\}$$

$$\sigma(na) - a(n\sigma) + \sigma(n'a) - a(n'\sigma)$$

$$- 2a(na)$$

$$\left. \begin{array}{l} a_1 n_1 + i a_1 n_2 \alpha_3 - i a_1 n_3 \alpha_2 \\ + i a_2 n_1 \alpha_3 + a_2 n_2 + i a_2 n_3 \alpha_1 \\ + i a_3 n_1 \alpha_2 - i a_3 n_2 \alpha_1 + a_3 n_3 \end{array} \right\} \alpha_1 \left\{ \begin{array}{l} \alpha_1 a_1 n_1' - i a_1 n_2' \alpha_3 + i a_1 n_3' \alpha_2 \\ + i a_2 n_1' \alpha_3 + a_2 n_2' - i a_2 n_3' \alpha_1 \\ + i a_3 n_1' \alpha_2 + i a_3 n_2' \alpha_1 + a_3 n_3' \end{array} \right\}$$

$$= \alpha_1 (a n') + \alpha_2 [n a]_3 - \alpha_3 [n a]_2 - i \rho_1 [n a] + \alpha_2 [n' a]_3 - \alpha_3 [n' a]_2 + i \rho_1 [n' a]$$

$$= \alpha_1 (a n') + [\alpha [n a]] + [\alpha [n' a]] - i \rho_1 [n a] + i \rho_1 [n' a]$$

$$= \rho_1 \left\{ (a n') \alpha + [\alpha [n a]] + [\alpha [n' a]] - i [n a] + i [n' a] \right\}$$

$$\begin{aligned}
& m c (\alpha a) \beta \alpha - m c \alpha \beta (\alpha a) \\
&= m c \left\{ \gamma' (\alpha, a_1 + \alpha_2 a_2 + \alpha_3 a_3) \overset{+i\beta_2}{\beta_3} \overset{-i\beta_2}{\beta_1} \alpha \sigma_1 - \gamma \beta_1 \alpha_1 \beta_3 (\alpha, a_1 + \alpha_2 a_2 + \alpha_3 a_3) \right\} \\
&= m c \left\{ i m c \left\{ \gamma' (a_1 \beta_1 - i a_2 \alpha_3 + i a_3 \alpha_2) \beta_2 + \gamma \beta_2 (\beta_1 a_1 + i a_2 \alpha_3 - i a_3 \alpha_2) \right\} \right. \\
&\quad \left. = \cancel{m c \left\{ a_1 [\alpha \alpha], \beta_2 \sigma_1 + \beta_2 [\alpha \alpha], \right\}} \right\} \rightarrow m c \left\{ [\alpha \alpha], \beta_1 \beta_2 - \beta_2 \beta_1 [\alpha \alpha], \right\} \\
&\quad = i m c \beta_3 [\alpha \alpha] \\
&= i m c \left\{ -i \beta_3 a_1 (\gamma - \gamma') - i [\alpha \alpha], \beta_2 \gamma' + i \beta_2 [\alpha \alpha] \gamma \right\} \\
&= m c \left\{ \beta_3 a_1 (\gamma - \gamma') + i \beta_3 [\alpha \alpha], (\gamma' + \gamma) \right\}
\end{aligned}$$

$$\varphi(-p_0 + e'A_0) + \varphi(\alpha, -p + e'A) + \beta_3 mc = 0$$

$$\varphi^* (p_0 - e'A_0 + (\alpha, -p + e'A) + \beta_3 mc) = 0$$

$$\varphi \left[-(p_0 - e'A_0)^2 + (\alpha, -p + e'A)^2 + mc^2 + (-p_0 + e'A_0)(\alpha, -p + e'A) + (\alpha, -p + e'A)(p_0 - e'A_0) \right]$$

$$(\alpha, -p + e'A)(\alpha, -p + e'A) = (-p + e'A)^2 + i \sum \alpha_3 (-p_1 + e'A_1)(-p_2 + e'A_2) - (-p_2 + e'A_2)(-p_1 + e'A_1)$$

$$e' \left(-p_1 A_2 - A_1 p_2 + p_2 A_1 + A_2 p_1 \right) = ie'h \left(-\frac{\partial A_2}{\partial x_1} + \frac{\partial A_1}{\partial x_2} \right) = -ie'h H_3$$

$$= (-p + e'A)^2 + e'h \sum \alpha_3 H_3 = (-p + e'A)^2 + e'h (\alpha H)$$

$$(-p_0 + e'A_0) \{ \alpha_1 (-p_1 + e'A_1) + \alpha_2 (-p_2 + e'A_2) + \alpha_3 (-p_3 + e'A_3) \} +$$

$$+ \{ (p_0 - e'A_0) \}$$

$$e' \left\{ -p_0 A_1 \alpha_1 - p_0 A_2 \alpha_2 - p_0 A_3 \alpha_3 - A_0 p_1 \alpha_1 - A_0 p_2 \alpha_2 - A_0 p_3 \alpha_3 \right.$$

$$\left. + A_1 p_0 \alpha_1 + A_2 p_0 \alpha_2 + A_3 p_0 \alpha_3 + p_1 A_0 \alpha_1 + p_2 A_0 \alpha_2 + p_3 A_0 \alpha_3 \right\}$$

$$= e' \left\{ i\hbar \frac{\partial A_1}{\partial t} \alpha_1 + i\hbar \frac{\partial A_2}{\partial t} \alpha_2 + i\hbar \frac{\partial A_3}{\partial t} \alpha_3 + i\hbar \frac{\partial A_0}{\partial x_1} \alpha_1 + i\hbar \frac{\partial A_0}{\partial x_2} \alpha_2 + i\hbar \frac{\partial A_0}{\partial x_3} \alpha_3 \right\}$$

$$= -i\hbar e \left\{ \alpha_1 \left(-\frac{\partial A_1}{\partial t} - \frac{\partial A_0}{\partial x_1} \right) + \alpha_2 \left(-\frac{\partial A_2}{\partial t} - \frac{\partial A_0}{\partial x_2} \right) + \alpha_3 \left(-\frac{\partial A_3}{\partial t} - \frac{\partial A_0}{\partial x_3} \right) \right\}$$

$$= -i\hbar e \left\{ \alpha_1 E_1 + \alpha_2 E_2 + \alpha_3 E_3 \right\} = -i\hbar e (\alpha E) = -i\hbar e_f (\alpha E)$$

$$\varphi \left[-(p_0 - e'A_0)^2 + (-p + e'A)^2 + e'h (\alpha H) + i\hbar e_f (\alpha E) \right] = 0$$

$$\varphi \left[-(p_0 + e'A_0)^2 + (-p + e'A)^2 + e'(QH) + i\hbar e_f (PE) \right] = 0$$

$$\varphi \left[-p_0^2 + p^2 - e'pA - e'A p - e'(QH) + e'(PE) \right] = 0$$

$$\varphi \left[\frac{\hbar^2 \partial^2}{c^2 \partial t^2} - \sum \hbar^2 \frac{\partial^2}{\partial x_k^2} + 2i\hbar e (A \text{ grad}) + e'(QH) + e'(PE) \right] = 0$$

$$+ m^2 c^2$$

$$\mathcal{L} \left[\frac{\hbar^2}{c^2} \frac{\partial^2}{\partial t^2} - \sum \hbar^2 \frac{\partial^2}{\partial x_k^2} + m^2 c^2 \right] = -2i\hbar c' (A \text{grad}) \varphi + e' \varphi (QH) + e' \varphi (PE)$$

$$\mathcal{L} = \alpha (A \text{grad} \varphi) + \beta \varphi (QH) + \gamma \varphi (PE)$$

$$\text{grad} \{ \varphi (QH) \} = \text{grad} \varphi (QH) + \varphi \text{grad} (QH)$$

$$\Delta (\varphi (QH)) = \Delta \varphi (QH) + \varphi \Delta (QH) + 2 \text{grad} \varphi \text{grad} (QH)$$

$$\varphi = v e^{\frac{i}{\hbar} (Et - (mcr))}$$

$$H = H e^{i\nu(t - \frac{r}{c})} + \tilde{H} e^{-i\nu(t - \frac{r}{c})}$$

$$\varphi_H = \dots e^{\frac{i}{\hbar} (E_2 t - m_2 r)}$$

$$\text{grad} \varphi = -\frac{i}{\hbar} m c \varphi$$

$$\frac{\partial \varphi}{\partial t} = \frac{i}{\hbar} E \varphi$$

$$A \varphi = \dots e$$

$$\text{grad} (QH) = -\frac{i\nu}{c} n (QH)$$

$$\frac{\partial (QH)}{\partial t} = i\nu (QH)$$

$$\Delta (\varphi (QH)) = \Delta \varphi (QH) + \varphi \Delta (QH) - \frac{2\nu}{\hbar c} (nm) \varphi (QH)$$

$$\frac{\partial^2}{c^2 \partial t^2} \varphi (QH) = \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} (QH) + \varphi \frac{\partial^2 H}{c^2 \partial t^2} + 2 \frac{\partial \varphi}{c^2 \partial t} \left(Q \frac{\partial H}{\partial t} \right)$$

$$= \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} (QH) + \varphi \left(Q \frac{\partial^2 H}{c^2 \partial t^2} \right) - 2 \frac{\nu}{c^2 \hbar} E \varphi (QH)$$

$$\frac{\hbar^2}{c^2} \frac{\partial^2}{\partial t^2} \varphi (QH) - \hbar^2 \Delta (\varphi QH) = (-m^2 c^2) \varphi (QH) - \frac{2\hbar \nu}{c} \left(\frac{E}{c} - (nm) \right) \varphi (QH)$$

$$\beta (-\hbar^2 \square + m^2 c^2) \varphi (QH) = -\frac{2\hbar \nu}{c} \left(\frac{E}{c} - (nm) \right) \varphi (QH)$$

$$\gamma (-\hbar^2 \square + m^2 c^2) \varphi (PE) = -\frac{2\hbar \nu}{c} \left(\frac{E}{c} - (nm) \right) \varphi (PE)$$

$$A \text{grad} \varphi$$

$$= -\frac{i}{\hbar} (A m c) \varphi$$

$$\alpha (-\hbar^2 \square + m^2 c^2) (A \text{grad}) \varphi = -\frac{2\hbar \nu}{c} \left(\frac{E}{c} - (nm) \right) (A \text{grad}) \varphi$$

$$-\frac{i}{\hbar} \times 2i\hbar = 2$$

$$\therefore -\alpha \cdot \frac{2\hbar \nu}{c} \left(\frac{E}{c} - (nm) \right) = -2i\hbar \frac{e}{\hbar} \quad \therefore \alpha = \frac{2i\hbar e}{2\hbar \nu \left(\frac{E}{c} - (nm) \right)} = 2i\hbar$$

$$-\beta \dots = -\frac{e}{c}$$

$$\beta = \frac{+e}{c}$$

$$-\gamma \dots = +\frac{e}{c}$$

$$\gamma = \frac{-e}{c}$$

$$\therefore \mathcal{L} = G_2 = K (2(mcA) + (QH) - (PE))$$

$$J = \alpha (\tilde{A} \text{grad} \varphi) + \beta \varphi(Q\tilde{H}) + \gamma \varphi(P\tilde{E}).$$

$$\text{grad} \varphi = -\frac{i}{\hbar} \nabla \varphi$$

$$\frac{\partial \varphi}{\partial t} = \frac{i}{\hbar} E \varphi$$

$$\text{grad}(Q\tilde{H}) = \frac{i\nu}{c} (Q\tilde{H})$$

$$\frac{\partial(Q\tilde{H})}{\partial t} = -i\nu(Q\tilde{H}).$$

$$\therefore \Delta \{ \varphi(Q\tilde{H}) \} = \Delta \varphi \cdot (Q\tilde{H}) + \varphi \cdot \Delta(Q\tilde{H}) + 2(n\hbar) \frac{\nu}{\hbar c} \varphi(Q\tilde{H}).$$

$$\frac{\partial^2}{\partial t^2} \varphi(Q\tilde{H}) = \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} (Q\tilde{H}) + \varphi \frac{\partial^2}{\partial t^2} (Q\tilde{H}) + \frac{2\nu}{c^2 \hbar} E \varphi(Q\tilde{H}).$$

$$-\hbar^2 \square \varphi(Q\tilde{H}) = -m^2 c^2 \varphi(Q\tilde{H}) + \left(\frac{2\nu \hbar}{c^2} E \varphi - \frac{2\nu \hbar}{c^2} \right) \varphi(Q\tilde{H}).$$

$$\beta (-\hbar^2 \square \varphi(Q\tilde{H}) + m^2 c^2 \varphi(Q\tilde{H})) = \beta \frac{2\nu \hbar}{c^2} \left(\frac{E}{c} - (m\hbar) \right) \varphi(Q\tilde{H}).$$

$$\beta (-\hbar^2 \square + m^2 c^2) (P\tilde{E}) = \beta \frac{2\nu \hbar}{c^2} \left(\frac{E}{c} - (m\hbar) \right) \varphi(P\tilde{E}).$$

$$\alpha (\quad) (\tilde{A} \text{grad} \varphi) = \alpha \frac{2\nu \hbar}{c^2} (\quad) (\tilde{A} \text{grad} \varphi). \quad \frac{-i}{\hbar} \nabla \varphi.$$

$$\alpha \frac{2\nu \hbar}{c^2} (\quad) = -2i\hbar \frac{e}{\hbar}.$$

$$\therefore \alpha = \frac{2i\hbar e}{-2(\tilde{A} m\hbar)}.$$

$$\beta \dots = -\frac{e}{c}$$

$$\beta = -K$$

$$\gamma = +K$$

$$\therefore G_1 = -K \{ 2(\tilde{A} m\hbar) + (Q\tilde{H}) \tilde{\Phi}(P\tilde{E}) \}.$$

$$V \left[\frac{E}{c} + m(\alpha m\hbar) + \beta_3 m c \right] \stackrel{(\alpha \tilde{A})}{=} 0.$$

$$V \left[\frac{E}{c} \right] \stackrel{(\alpha \tilde{A})}{=} (\alpha, m_1 + \alpha_2 m_2 + \alpha_3 m_3) (\alpha, \tilde{A}_1 + \alpha_2 \tilde{A}_2 + \alpha_3 \tilde{A}_3)$$

$$= -\alpha_1 \tilde{A}_1 (\alpha, m_1 + \alpha_2 m_2 + \alpha_3 m_3) + 2 \tilde{A}_1 m_1$$

$$(\alpha m\hbar) \stackrel{(\alpha \tilde{A})}{=} -(\alpha, \tilde{A}_1) (\alpha m\hbar) + 2 (\tilde{A}_1 m\hbar).$$

$$V(\alpha \tilde{A}) \left[\frac{E}{c} \tilde{\Phi}(\alpha \mathcal{M}) - \beta_3 m c \right] = -2 V(\tilde{A} \mathcal{M}).$$

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$$G_1 = -K V(\alpha \tilde{A}) \left[-\frac{E_1}{c} + (\alpha \mathcal{M}_1) + \beta_3 m c \right]$$

$$= K V(\alpha \tilde{A}) \left[\frac{E_1}{c} - (\alpha \mathcal{M}_1) - \beta_3 m c \right] = K V(\alpha \tilde{A}) \left[\frac{E}{c} - (\alpha \mathcal{M}) - \beta_3 m c \right. \\ \left. - \frac{h\nu}{c} + \frac{h\nu}{c} (\alpha n) \right]$$

$$= K V \left\{ -2 V(\tilde{A} \mathcal{M}) - \frac{h\nu}{c} (\alpha \tilde{A}) + \frac{h\nu}{c} (\alpha \tilde{A}) (\alpha n) \right\}$$

$$= -K V \left\{ (\tilde{A} \mathcal{M}) + \frac{h\nu}{c} (\alpha \tilde{A}) - i \frac{h\nu}{c} (\tilde{\mathcal{M}}, A \times n) \right\}$$

$$= -K V \left\{ (\tilde{A} \mathcal{M}) - i h (\alpha \tilde{E}) + h (\alpha, H) \right\} = -K V \left\{ (\tilde{A} \mathcal{M}) + (QH) - (PE) \right\}$$

$$\frac{i\nu}{c} \tilde{A} = \tilde{E} \quad \therefore \frac{\nu}{c} \tilde{A} = -i \tilde{E} \quad \frac{i\nu}{c} [n \times A] = H$$

$$\left(-(p_0 - e'A_0) + (\alpha_1^+, -p + e'A) + \beta_3^+ m c \right) \varphi = 0$$

$$((p_0 - e'A) + (\alpha_1^+, -p + e'A) + \beta_3^+ m c)$$

$$\left[-(p_0 - e'A_0)^2 + (\alpha_1^+, -p + e'A)^2 + m^2 c^2 + (p_0 - e'A)(\alpha_1^+, -p + e'A) + (\alpha_1^+, -p + e'A)(-p_0 + e'A_0) \right] \varphi = 0$$

$$(\alpha_1^+, -p + e'A)^2 = (-p + e'A)^2 + i \sum \alpha_3^+ (\quad) = (-p + e'A)^2 - e' h (\alpha_1^+ H) \\ \downarrow -i h e \rho_1^+ (\alpha_1^+ E)$$

$$\rho_1^+ \alpha_1^+ = \tilde{\alpha}_1 \rho_1$$

$$\rho_2^+ \alpha_2^+ = \tilde{\alpha}_2 \rho_2$$

$$\therefore \rho_1^+ (\alpha_1^+ E) = \overline{(\alpha_1^+ E)} \rho_1^+$$